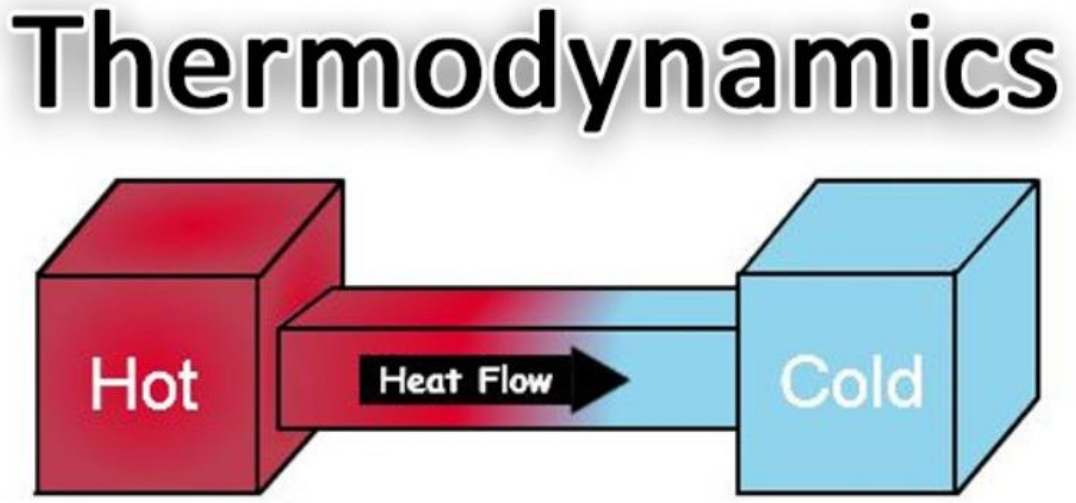
DEPARTMENT OF MECHANICAL ENGINEERING

ABIT

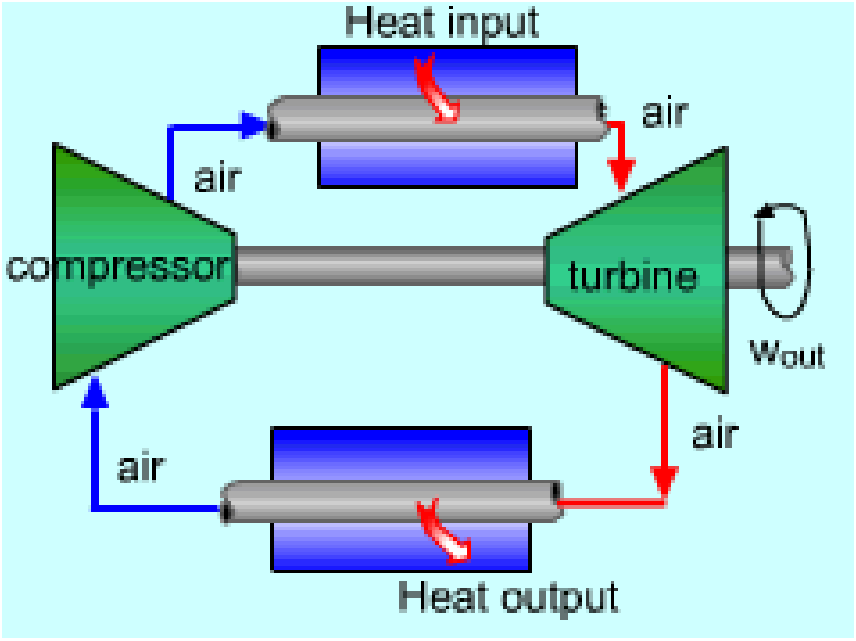
ENGINEERING THERMODYNAMICS



MODULE-IV

Gas Power Cycles

PETROL ENGINE **DIESEL ENGINE**



DEFINITION OF A CYCLE

- A cycle is defined as a repeated series of operations occurring in a certain order. It may be repeated by repeating the processes in the same order. The cycle may be of imaginary perfect engine or actual engine. The former is called ideal cycle and the latter actual cycle. In ideal cycle all accidental heat losses are prevented and the working substance is assumed to behave like a perfect working substance.

AIR STANDARD EFFICIENCY

- To compare the effects of different cycles, it is of paramount importance that the effect of the calorific value of the fuel is altogether eliminated and this can be achieved by considering air (which is assumed to behave as a perfect gas) as the working substance in the engine cylinder. The efficiency of engine using air as the working medium is known as an “Air standard efficiency”. This efficiency is oftenly called ideal efficiency.

- The actual efficiency of a cycle is always less than the air-standard efficiency of that cycle under ideal conditions.

- This is taken into account by introducing a new term “Relative efficiency” which is defined as :

  $\eta_{relative}$=Actual thermal efficiency/Air standard efficiency

- The analysis of all air standard cycles is based upon the following assumptions :

  Assumptions :

  1. The gas in the engine cylinder is a perfect gas i.e., it obeys the gas laws and has constant specific heats.

  2. The physical constants of the gas in the cylinder are the same as those of air at moderate temperatures i.e., the molecular weight of cylinder gas is 29. cp = 1.005 kJ/kg-K, cv = 0.718 kJ/kg-K.

  3. The compression and expansion processes are adiabatic and they take place without internal friction, i.e., these processes are isentropic.

  4. No chemical reaction takes place in the cylinder. Heat is supplied or rejected by bringing a hot body or a cold body in contact with cylinder at appropriate points during the process.

  5. The cycle is considered closed with the same ‘air’ always remaining in the cylinder to repeat the cycle.

THE CARNOT CYCLE

This cycle has the highest possible efficiency and consists of four simple operations namely,

(a) Isothermal expansion

(b) Adiabatic expansion

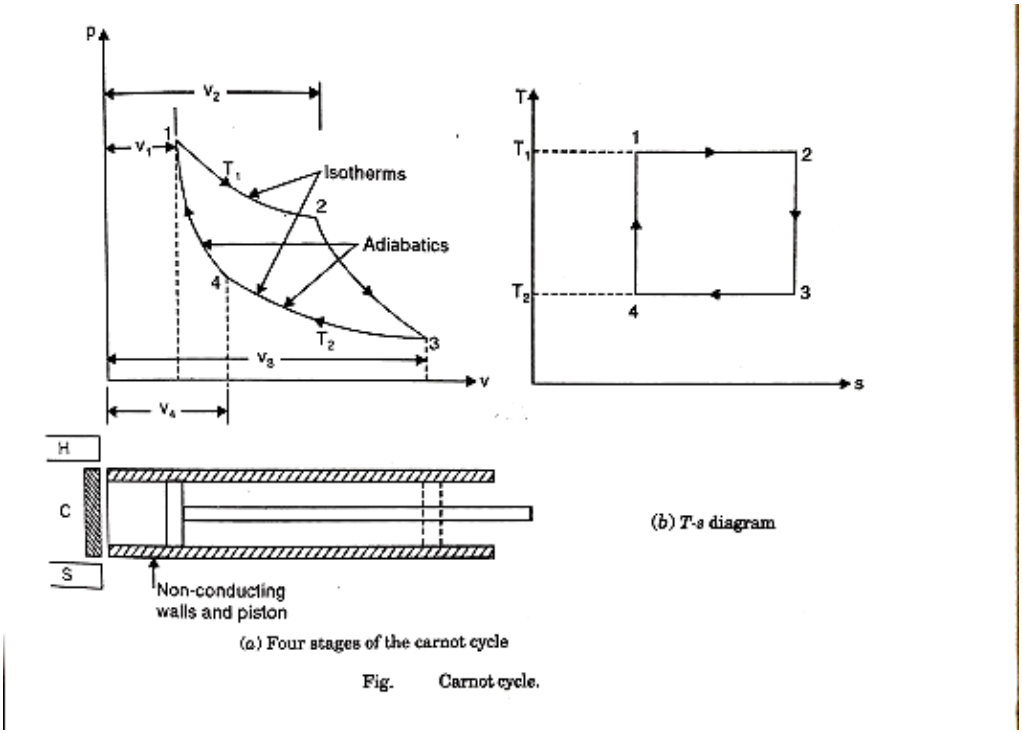
(c) Isothermal compression

(d) Adiabatic compression.

The condition of the Carnot cycle may be imagined to occur in the following way :

- One kg of a air is enclosed in the cylinder which (except at the end) is made of perfect nonconducting material. A source of heat 'H' is supposed to provide unlimited quantity of heat, nonconducting cover 'C' and a sump 'S' which is of infinite capacity so that its temperature remains unchanged irrespective of the fact how much heat is supplied to it. The temperature of source H is T1 and the same is of the working substance. The working substance while rejecting heat to sump 'S' has the temperature. T2 i.e., the same as that of sump S.

Following are the four stages of the Carnot cycle.



(a) Four stages of the carnot cycle

(b) T-s diagram

Fig. Carnot cycle.

Stage (1). Line 1-2 [Fig. 13.1 (a)] represents the isothermal expansion which takes place at temperature T1 when source of heat H is applied to the end of cylinder. Heat supplied in this case is given by RT1 loge r and where r is the ratio of expansion.

Stage (2). Line 2-3 represents the application of non-conducting cover to the end of the cylinder. This is followed by the adiabatic expansion and the temperature falls from T1 to T2.

Stage (3). Line 3-4 represents the isothermal compression which takes place when sump 'S' is applied to the end of cylinder. Heat is rejected during this operation whose value is given by RT2 loge r where r is the ratio of compression.

Stage (4). Line 4-1 represents repeated application of non-conducting cover and adiabatic compression due to which temperature increases from T2 to T1.

It may be noted that ratio of expansion during isotherm 1-2 and ratio of compression during isotherm 3-4 must be equal to get a closed cycle. Fig.(b) represents the Carnot cycle on T-s coordinates.

Now according to law of conservation of energy,

Heat supplied = Work done + Heat rejected

Work done = Heat supplied – Heat rejected

= RT1 . loge r – RT2 loge r

Efficiency of cycle=work done/heat supplied= $\frac{R\log_e r(T_1 - T_2)}{RT_1 \log_e r} = \frac{T_1 - T_2}{T_1}$

Example.

A Carnot engine working between 400°C and 40°C produces 130 kJ of work. Determine :

(i) The engine thermal efficiency.

(ii) The heat added.

(iii) The entropy changes during heat rejection process.

Solution.

Temperature, T1 = T2 = 400 + 273 = 673 K

Temperature, T3 = T4 = 40 + 273 = 313 K

Work produced, W = 130 kJ.

(i) Engine thermal efficiency, η_{th}:

$$\eta_{th} = \frac{673 - 313}{673} = 0.535 = 53.5\%$$

(ii) Heat added :

$$\eta_{th} = \frac{workdone}{heatadded}$$

$$0.535 = \frac{130}{heatadded}$$

Heat added=243 KJ

(iii) Entropy change during the heat rejection process, (S3 – S4) :

Heat rejected = Heat added – Work done = 243 – 130 = 113 kJ

Heat rejected = T3 (S3 – S4) = 113

∴ (S3 – S4) =113/T_2=113/313=0.361 KJ/K

Example .

0.5 kg of air (ideal gas) executes a Carnot power cycle having a thermal efficiency of 50 per cent. The heat transfer to the air during the isothermal expansion is 40 kJ. At the beginning of the isothermal expansion the pressure is 7 bar and the volume is 0.12 m^3. Determine : (i) The maximum and minimum temperatures for the cycle in K ; (ii) The volume at the end of isothermal expansion in m^3 ; (iii) The heat transfer for each of the four processes in kJ. For air c_V = 0.721 kJ/kg K, and c_P = 1.008 kJ/kg K.

Solution. Given : m = 0.5 kg ; η_{th} = 50% ; Heat transferred during isothermal expansion = 40 kJ ; p1 = 7 bar, V1 = 0.12 m3 ; c_v= 0.721 kJ/kg K ; c_p = 1.008 kJ/kg K.

(i) The maximum and minimum temperatures, T_1, T_2 :

$p_1V_1 = mRT_1$

$7 \times 105 \times 0.12 = 0.5 \times 287 \times T_1$

∴ Maximum temperature, T_1=585.4K

$$\eta_{cycle} = \frac{T_1 - T_2}{T_1} \Rightarrow 0.5 = \frac{585.4 - T_2}{585.4}$$

∴ Minimum temperature, T2 = 585.4 – 0.5 × 585.4 = 292.7 K.

(ii) The volume at the end of isothermal expansion, V_2:

Heat transferred during isothermal expansion

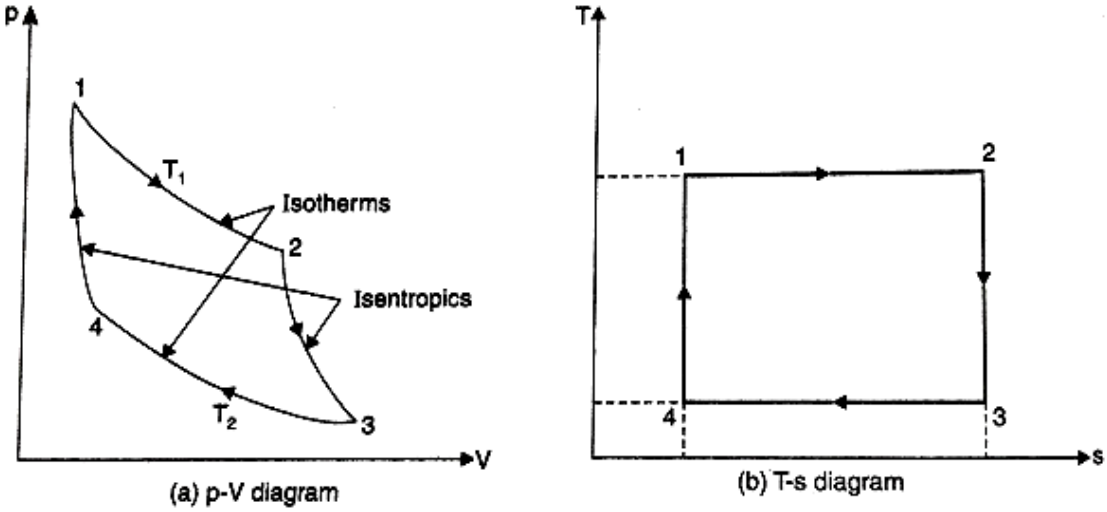


Fig. Carnot cycle.

$$= P_1V_1 \ln r = mRT_1 \ln\left(\frac{V_2}{V_1}\right) = 40X10^3$$

$$\Rightarrow 0.5X\,287\,X\,585.4\ln\left(\frac{V_2}{0.12}\right) = 40X10^3$$

Or, V_2 = 0.193 m3.

(iii) The heat transfer for each of the four processes :

Process	Classification	Heat transfer
1-2	Isothermal expansion	40KJ
2-3	Adiabatic reversible expansion	zero
3-4	Isothermal compression	-40KJ
4-1	Adiabatic reversible compression	zero

CONSTANT VOLUME OR OTTO CYCLE

- This cycle is so named as it was conceived by 'Otto'. On this cycle, petrol, gas and many types of oil engines work. It is the standard of comparison for internal combustion engines.
- Figs. (a) and (b) shows the theoretical p-V diagram and T-s diagrams of this cycle respectively.
- The point 1 represents that cylinder is full of air with volume V_1, pressure p_1 and absolute temperature T_1.
- Line 1-2 represents the adiabatic compression of air due to which p_1, V_1 and T_1 change to p_2, V_2 and T_2, respectively.
- Line 2-3 shows the supply of heat to the air at constant volume so that p_2 and T_2 change to p_3 and T_3 (V_3 being the same as V_2).
- Line 3-4 represents the adiabatic expansion of the air. During expansion p_3, V_3 and T_3 change to a final value of p_4, V_4 or V_1 and T_4, respectively.

- Line 4-1 shows the rejection of heat by air at constant volume till original state (point 1) reaches.

Consider 1 kg of air (working substance):

Heat supplied at constant volume = $c_v(T_3 - T_2)$.

Heat rejected at constant volume = $c_v (T_4 - T_1)$.

But, work done = Heat supplied – Heat rejected = $c_v (T_3 - T_2) - c_v (T_4 - T_1)$

$$Efficiency = \frac{Workdone}{Heat\ \sup\ plied} = \frac{C_V(T_3 - T_2) - C_V(T_4 - T_1)}{C_V(T_3 - T_2)} = 1 - \frac{T_4 - T_1}{T_3 - T_2} .....(i)$$

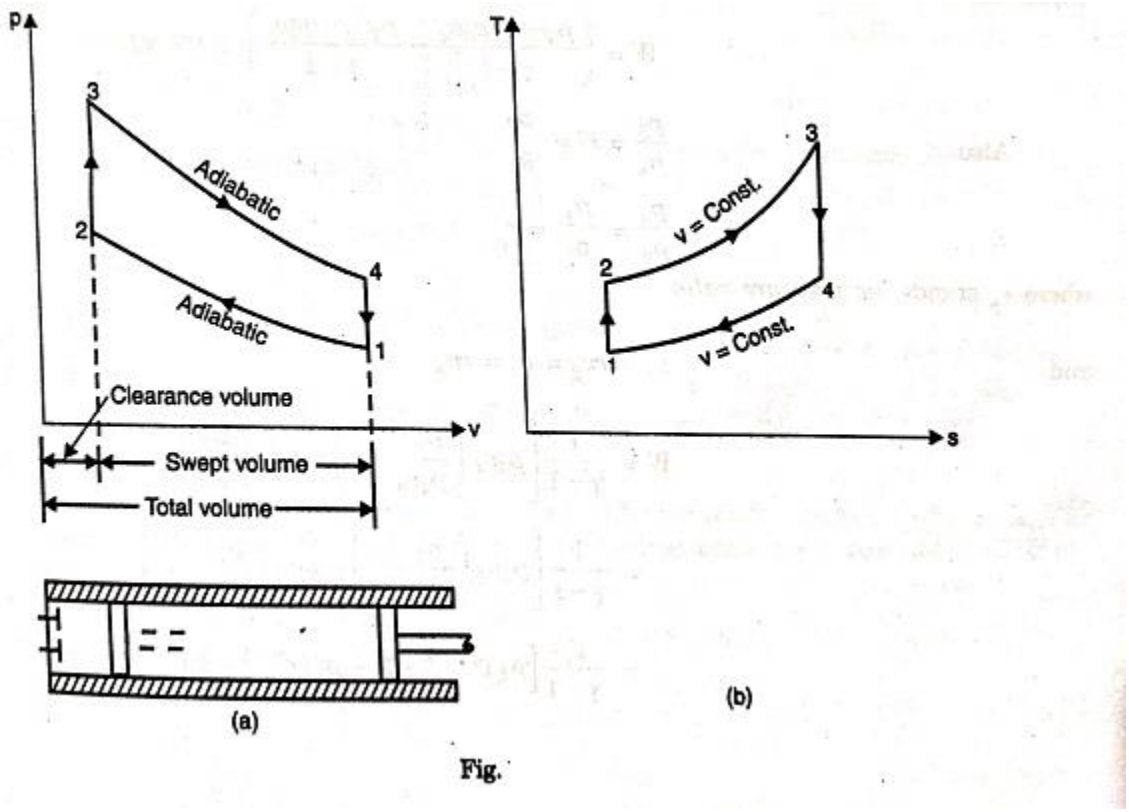


Fig.

Let compression ratio, $r_c = r = \frac{v_1}{v_2}$

And expansion raio, $r_e = r = \frac{v_4}{v_3}$

(These two ratios are same in this cycle)

As $\frac{T_2}{T_1}=\left(\frac{v_1}{v_2}\right)^{\gamma-1} \Rightarrow T_2=T_1(r)^{\gamma-1}$

Similarly $\frac{T_3}{T_4}=\left(\frac{v_4}{v_3}\right)^{\gamma-1} \Rightarrow T_3=T_4(r)^{\gamma-1}$

Inserting the values of T_2 and T_3 in equation (i), we get

$$\eta_{Otto}=1-\frac{T_4-T_1}{T_4(r)^{\gamma-1}-T_1(r)^{\gamma-1}}=1-\frac{T_4-T_1}{r^{\gamma-1}(T_4-T_1)}=1-\frac{1}{r^{\gamma-1}}$$

This expression is known as the air standard efficiency of the Otto cycle.

- It is clear from the above expression that efficiency increases with the increase in the value of r, which means we can have maximum efficiency by increasing r to a considerable extent, but due to practical difficulties its value is limited to about 8.
- The net work done per kg in the Otto cycle can also be expressed in terms of p, v. If p is expressed in bar i.e., 105 N/m2, then work done

$$W=\left(\frac{p_3v_3-p_4v_4}{\gamma-1}-\frac{p_2v_2-p_1v_1}{\gamma-1}\right)X10^2\,KJ$$

Also, $\frac{p_3}{p_4}=r^{\gamma}=\frac{p_2}{p_1}$

Hence, $\frac{p_3}{p_2}=r^{p}=\frac{p_4}{p_1}$

where rp stands for pressure ratio.

And $v_1 = rv_2 = v_4 = rv_3$ $\left[\frac{v_1}{v_2}=\frac{v_4}{v_3}=r\right]$

$$W=\frac{1}{\gamma-1}\left[p_4v_4\left(\frac{p_3v_3}{p_4v_4}-1\right)-p_1v_1\left(\frac{p_2v_2}{p_1v_1}-1\right)\right]$$

$$=\frac{1}{\gamma-1}\left[p_4v_4\left(\frac{p_3}{p_4r}-1\right)-p_1v_1\left(\frac{p_2}{p_1r}-1\right)\right]$$

$$= \frac{v_1}{\gamma - 1}\left[p_4(r^{\gamma-1} - 1) - p_1(r^{\gamma-1} - 1)\right]$$

$$= \frac{v_1}{\gamma - 1}\left[(r^{\gamma-1} - 1)(p_4 - 1)\right]$$

$$= \frac{p_1 v_1}{\gamma - 1}\left[(r^{\gamma-1} - 1)(r_p - 1)\right]$$

Mean effective pressure (pm) is given by :

$$p_m = \left[\left(\frac{p_3 v_3 - p_4 v_4}{\gamma - 1} - \frac{p_2 v_2 - p_1 v_1}{\gamma - 1}\right)/(v_1 - v_2)\right] bar$$

Also, $$p_m = \frac{\left[\frac{p_1 v_1}{\gamma - 1}\left[(r^{\gamma-1} - 1)(r_p - 1)\right]\right]}{v_1 - v_2}$$

$$= \frac{\frac{p_1 v_1}{\gamma - 1}\left[(r^{\gamma-1} - 1)(r_p - 1)\right]}{v_1 - \frac{v_1}{r}}$$

$$= \frac{\frac{p_1 v_1}{\gamma - 1}\left[(r^{\gamma-1} - 1)(r_p - 1)\right]}{v_1\left(\frac{r-1}{r}\right)}$$

$$\Rightarrow p_m = \frac{p_1 r\left[(r^{\gamma-1} - 1)(r_p - 1)\right]}{(\gamma - 1)(r - 1)}$$

Example. The efficiency of an Otto cycle is 60% and γ = 1.5. What is the compression ratio ?

Solution. Efficiency of Otto cycle, η = 60% Ratio of specific heats, γ = 1.5 Compression ratio, r = ?
Efficiency of Otto cycle is given by,

$$\eta_{Otto} = 1 - \frac{1}{(r)^{\gamma-1}}$$

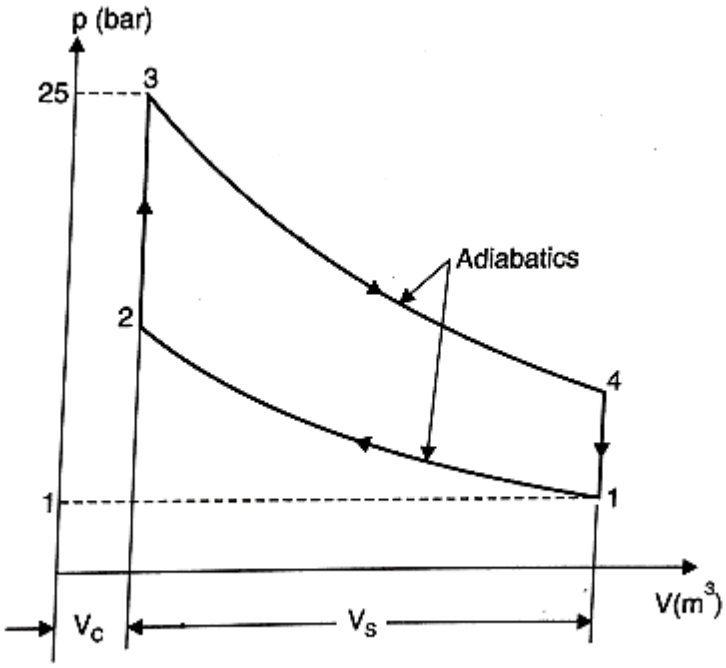
$$0.6 = 1 - \frac{1}{(r)^{1.5-1}}$$

$$\Rightarrow \frac{1}{(r)^{0.5}} = 0.4 \Rightarrow r = 6.25$$

Hence, compression ratio=6.25.

Example. An engine of 250 mm bore and 375 mm stroke works on Otto cycle. The clearance volume is 0.00263 m3. The initial pressure and temperature are 1 bar and 50°C. If the maximum pressure is limited to 25 bar, find the following : (i) The air standard efficiency of the cycle. (ii) The mean effective pressure for the cycle. Assume the ideal conditions.

Solution. Bore of the engine, D = 250 mm = 0.25 m Stroke of the engine, L = 375 mm = 0.375 m Clearance volume, Vc = 0.00263 m3 Initial pressure, p1 = 1 bar Initial temperature, T1 = 50 + 273 = 323 K



Fig

Maximum pressure, p_3 = 25 bar Swept volume, $V_s = \pi/4\ D^2L = \pi/4 \times 0.25^2 \times 0.375 = 0.0184$ m3

Compression ratio, $r = \frac{V_S + V_C}{V_C} = \frac{0.0184 + 0.00263}{0.00263} = 8$

(i) Air standard efficiency : The air standard efficiency of Otto cycle is given by

$$\eta_{Otto} = 1 - \frac{1}{(r)^{\gamma-1}} = 1 - \frac{1}{(8)^{1.4-1}} = 0.565 or 56.5\%$$

(ii) Mean effective pressure, p_m : For adiabatic (or isentropic) process 1-2

$$p_1 v_1^{\gamma} = p_2 v_2^{\gamma} \Rightarrow p_2 = p_1 \left(\frac{v_1}{v_2}\right)^{\gamma} = 1X(r)^{1.4} = 1X(8)^{1.4} = 18.38 bar$$

$\therefore$ Pressure ratio, $r_p = \frac{p_3}{p_2} = \frac{25}{18.38} = 1.36$

The mean effective pressure is given by

$$\Rightarrow p_m = \frac{p_1 r\left[\left(r^{\gamma-1} - 1\right)(r_p - 1)\right]}{(\gamma - 1)(r - 1)} = \frac{1x8\left[(8)^{1.4-1} - 1)(1.36 - 1)\right]}{(1.4 - 1)(8 - 1)} = 1.334 bar$$

CONSTANT PRESSURE OR DIESEL CYCLE

- This cycle was introduced by Dr. R. Diesel in 1897. It differs from Otto cycle in that heat is supplied at constant pressure instead of at constant volume. Fig. (a and b) shows the p-v and T-s diagrams of this cycle respectively.
- This cycle comprises of the following operations :

  (i) 1-2.....Adiabatic compression.

  (ii) 2-3.....Addition of heat at constant pressure.

  (iii) 3-4.....Adiabatic expansion.

  (iv) 4-1.....Rejection of heat at constant volume.

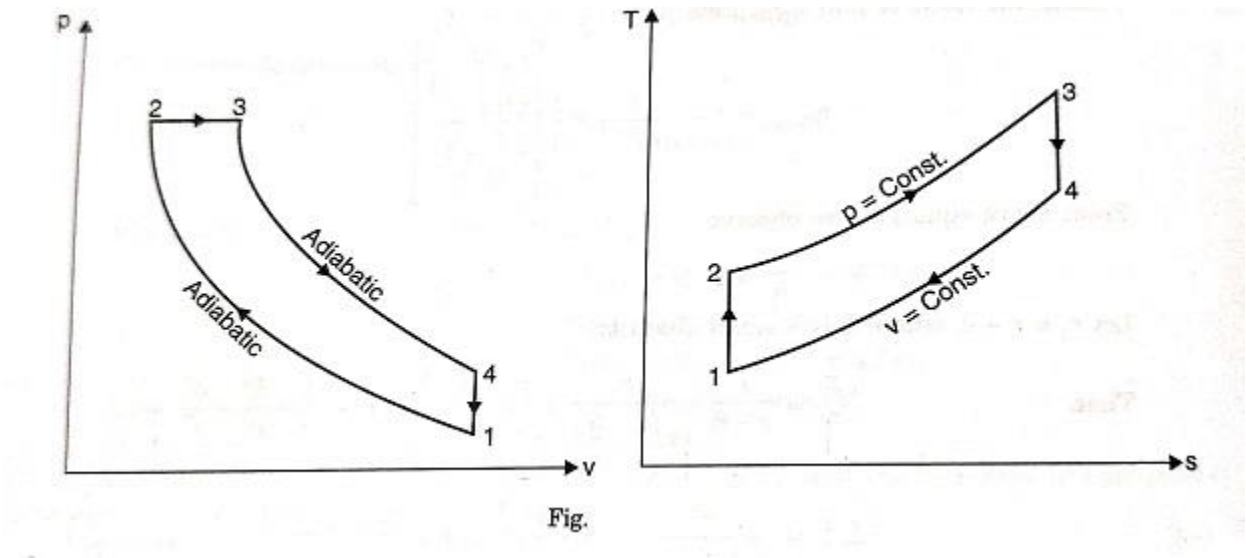


Fig.

- Point 1 represents that the cylinder is full of air. Let p1, V1 and T1 be the corresponding pressure, volume and absolute temperature. The piston then compresses the air adiabatically (i.e., pV^{γ} = constant) till the values become p2, V2 and T2 respectively (at the end of the stroke) at point 2. Heat is then added from a hot body at a constant pressure.
- During this addition of heat let volume increases from V2 to V3 and temperature T2 to T3, corresponding to point 3. This point (3) is called the point of cut-off. The air then expands adiabatically to the conditions p4, V4 and T4 respectively corresponding to point 4. Finally, the air rejects the heat to the cold body at constant volume till the point 1 where it returns to its original state.
- Consider 1 kg of air.
- Heat supplied at constant pressure = $c_p(T_3 - T_2)$
- Heat rejected at constant volume = $c_v(T_4 - T_1)$
- Work done = Heat supplied – heat rejected = $c_p(T_3 - T_2) - c_v(T_4 - T_1)$
- $\eta_{diesel} = \frac{workdone}{heat\ \sup\ plied} = \frac{c_p(T_3 - T_2) - c_v(T_4 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{(T_4 - T_1)}{\gamma(T_3 - T_2)} \ldots\ldots\ldots\ldots(i) \left[\because \frac{c_p}{c_v} = \gamma\right]$
- Let compression ratio, $r = \frac{v_1}{v_2}$ and cut-off ratio, $\rho = \frac{v_3}{v_2} i.e, \frac{volumeatcutoff}{clearancevolume}$

Now, during adiabatic compression 1-2,

$$\frac{T_2}{T_1}=\left(\frac{v_1}{v_2}\right)^{\gamma-1}=r^{\gamma-1} or T_2=T_1(r)^{\gamma-1}$$

During constant pressure process 2-3,

$$\frac{T_3}{T_2}=\frac{v_3}{v_2}=\rho \Rightarrow T_3=\rho T_2=\rho T_1(r)^{\gamma-1}$$

During adiabatic expansion 3-4

$$\frac{T_3}{T_4}=\left(\frac{v_4}{v_3}\right)^{\gamma-1}=\left(\frac{r}{\rho}\right)^{\gamma-1}\left(\because \frac{v_4}{v_3}=\frac{v_1}{v_3}=\frac{v_1}{v_2}\,X\,\frac{v_2}{v_3}=\frac{r}{\rho}\right)$$

$$\therefore T_4=\frac{T_3}{\left(\frac{r}{\rho}\right)^{\gamma-1}}=\frac{\rho T_1(r)^{\gamma-1}}{\left(\frac{r}{\rho}\right)^{\gamma-1}}=T_1\rho^{\gamma}$$

By inserting values of T2, T3 and T4 in eqn. (i), we get

$$\eta_{diesel}=1-\frac{(T_1\rho^{\gamma}-T_1)}{\gamma\left(\rho T_1(r)^{\gamma-1}-T_1(r)^{\gamma-1}\right)}=1-\frac{(\rho^{\gamma}-1)}{\gamma(r)^{\gamma-1}(\rho-1)} \Rightarrow \eta_{diesel}=1-\frac{1}{\gamma(r)^{\gamma-1}}\left[\frac{\rho^{\gamma}-1}{\rho-1}\right]$$

- It may be observed that eqn. for efficiency of diesel cycle is different from that of the Otto cycle only in bracketed factor. This factor is always greater than unity, because ρ > 1. Hence for a given compression ratio, the Otto cycle is more efficient.
- The net work for diesel cycle can be expressed in terms of pv as follows :

$$W=p_2(v_3-v_2)+\frac{p_3v_3-p_4v_4}{\gamma-1}-\frac{p_2v_2-p_1v_1}{\gamma-1}$$

$$=p_2(\rho v_2-v_2)+\frac{p_3\rho v_2-p_4rv_2}{\gamma-1}-\frac{p_2v_2-p_1rv_2}{\gamma-1}$$

$$\left[\because \frac{v_3}{v_2}=\rho \therefore v_3=\rho v_2 and\ \frac{v_1}{v_2}=r \therefore v_1=rv_2, but v_4=v_1 \therefore v_4=rv_2\right]$$

$$=p_2v_2(\rho-1)+\frac{p_3\rho v_2-p_4rv_2}{\gamma-1}-\frac{p_2v_2-p_1rv_2}{\gamma-1}$$

$$= \frac{v_2\left[p_2(\rho-1)(\gamma-1)+p_3\rho-p_4r-(p_2-p_1r)\right]}{\gamma-1}$$

$$= \frac{v_2\left[p_2(\rho-1)(\gamma-1)+p_3\left(\rho-\frac{p_4r}{p_3}\right)-p_2\left(1-\frac{p_1r}{p_2}\right)\right]}{\gamma-1}$$

$$= \frac{p_2v_2\left[(\rho-1)(\gamma-1)+\rho-\rho^{\gamma}r^{1-\gamma}-(1-r^{1-\gamma})\right]}{\gamma-1} \qquad \left[\because \frac{p_4}{p_3}=\left(\frac{v_3}{v_4}\right)^{\gamma}=\left(\frac{\rho}{r}\right)^{\gamma}=\rho^{\gamma}r^{-\gamma}\right]$$

$$= \frac{p_1v_1r^{\gamma-1}\left[(\rho-1)(\gamma-1)+\rho-\rho^{\gamma}r^{1-\gamma}-(1-r^{1-\lambda})\right]}{\gamma-1}$$

$$\left[\because \frac{p_2}{p_1}=\left(\frac{v_1}{v_2}\right)^{\gamma} or p_2=p_1r^{\gamma} and \frac{v_1}{v_2}=ror v_2=v_1r^{-\gamma}\right]$$

$$= \frac{p_1v_1r^{\gamma-1}\left[\gamma(\rho-1)-r^{1-\gamma}(\rho^{\gamma}-1)\right]}{(\gamma-1)}$$

Mean effective pressure pm is given by :

$$p_m = \frac{p_1v_1r^{\gamma-1}\left[\gamma(\rho-1)-r^{1-\gamma}(\rho^{\gamma}-1)\right]}{(\gamma-1)v_1\left(\frac{r-1}{r}\right)}$$

0r, $$p_m = \frac{p_1r^{\gamma}\left[\gamma(\rho-1)-r^{1-\gamma}(\rho^{\lambda}-1)\right]}{(\gamma-1)(r-1)}$$

Example.

A diesel engine has a compression ratio of 15 and heat addition at constant pressure takes place at 6% of stroke. Find the air standard efficiency of the engine. Take γ for air as 1.4.

Solution.

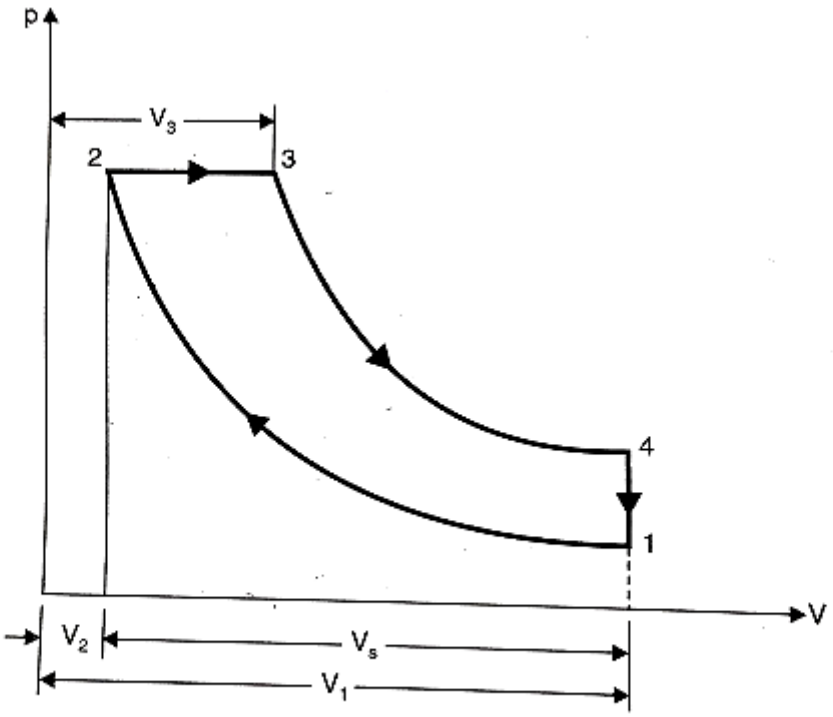


Fig.

Compression ratio, $r = \frac{v_1}{v_2} = 15$

γ for air = 1.4

Air standard efficiency of diesel cycle is given by

$$\eta_{diesel} = 1 - \frac{1}{\gamma (r)^{\gamma - 1}} \left[\frac{\rho^{\gamma} - 1}{\rho - 1} \right]$$

where ρ = cut-off ratio $= \frac{v_3}{v_2}$

But $v_3 - v_2 = \frac{6}{100} v_s$ (Vs = stroke volume)

= 0.06 ($V_1 - V_2$) = 0.06 (15 $V_2 - V_2$)

= 0.84 V_2 or $V_3 = 1.84\ V_2$

$$\rho = \frac{v_3}{v_2} = \frac{1.84 v_2}{v_2} = 1.84$$

So we get,

$$\eta_{diesel} = 1 - \frac{1}{1.4(15)^{1.4-1}} \left[\frac{(1.84)^{1.4} - 1}{1.84 - 1} \right] = 0.612 = 61.2\%$$

Example

. The stroke and cylinder diameter of a compression ignition engine are 250 mm and 150 mm respectively. If the clearance volume is 0.0004 m3 and fuel injection takes place at constant pressure for 5 per cent of the stroke determine the efficiency of the engine. Assume the engine working on the diesel cycle.

Solution.

Length of stroke, L = 250 mm = 0.25 m

Diameter of cylinder, D = 150 mm = 0.15 m

Clearance volume, V_2 = 0.0004 m3

Swept volume, $V_s = \pi/4\ D^2L = \pi/4 \times 0.15^2 \times 0.25 = 0.004418\ m^3$

Total cylinder volume = Swept volume + clearance volume = 0.004418 + 0.0004 = 0.004818 m^3

Volume at point of cut-off, $v_3 = v_2 + \frac{5}{100} v_s = 0.0004 + \frac{5}{100} x 0.004818 = 0.000621 m^3$

∴ Cut-off ratio, $\rho = \frac{v_3}{v_2} = \frac{0.000621}{0.0004} = 1.55$

Compression ratio, $$r = \frac{v_1}{v_2} = \frac{v_s + v_2}{v_2} = \frac{0.004818 + 0.0004}{0.0004} = 12.04$$

Hence, $$\eta_{diesel} = 1 - \frac{1}{\gamma (r)^{\gamma - 1}} \left[\frac{\rho^{\gamma} - 1}{\rho - 1} \right] = 1 - \frac{1}{1.4(12.04)^{1.4-1}} \left[\frac{(1.55)^{1.4} - 1}{1.55 - 1} \right] = 0.593 = 59.3\%$$

DUAL COMBUSTION CYCLE

- This cycle (also called the limited pressure cycle or mixed cycle) is a combination of Otto and Diesel cycles, in a way, that heat is added partly at constant volume and partly at constant pressure ; the advantage of which is that more time is available to fuel (which is injected into the engine cylinder before the end of compression stroke) for combustion. Because of lagging characteristics of fuel this cycle is invariably used for diesel and hot spot ignition engines.
- The dual combustion cycle consists of the following operations :

(i) 1-2—Adiabatic compression

(ii) 2-3—Addition of heat at constant volume

(iii) 3-4—Addition of heat at constant pressure

(iv) 4-5—Adiabatic expansion

(v) 5-1—Rejection of heat at constant volume.

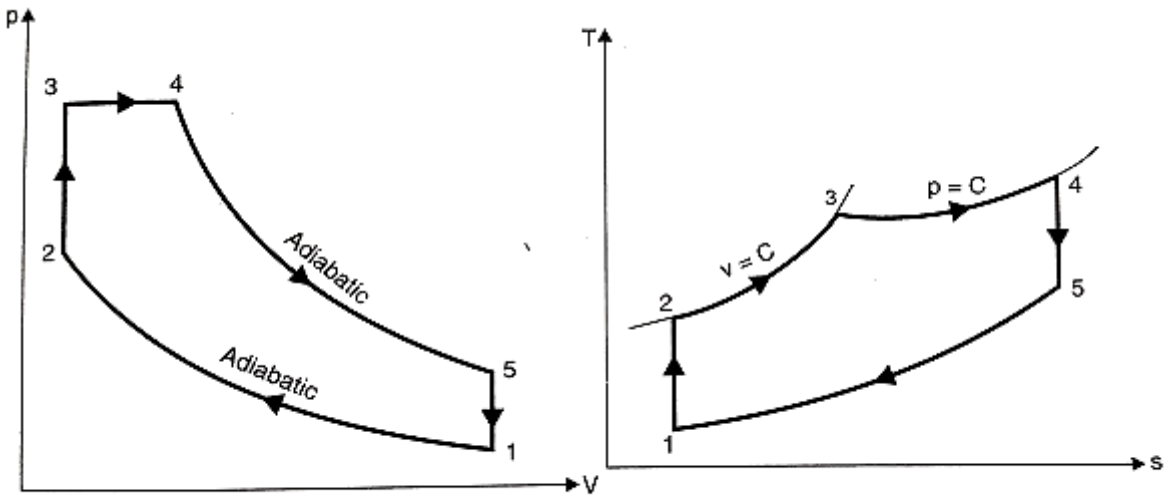


Fig.

Consider 1 kg of air.

Total heat supplied = Heat supplied during the operation 2-3 + heat supplied during the operation 3-4

= $c_v(T_3 - T_2) + c_p(T_4 - T_3)$

Heat rejected during operation 5-1 = $c_v(T_5 - T_1)$

Work done = Heat supplied – heat rejected

= $c_v(T_3 - T_2) + c_p(T_4 - T_3) - c_v(T_5 - T_1)$

$$\eta_{dual} = \frac{work done}{heat\ \sup\ plied} = \frac{c_v(T_3 - T_2) + c_p(T_4 - T_3) - c_v(T_5 - T_1)}{c_v(T_3 - T_2) + c_p(T_4 - T_3)}$$

$$= 1 - \frac{c_v(T_5 - T_1)}{c_v(T_3 - T_2) + c_p(T_4 - T_3)}$$

$$= 1 - \frac{(T_5 - T_1)}{(T_3 - T_2) + \gamma(T_4 - T_3)} \quad \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

Compression ratio, $r = \frac{v_1}{v_2}$

During adiabatic compression process 1-2,

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = (r)^{\gamma - 1} \quad \ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

During constant volume heating process,

$$\frac{p_3}{T_3} = \frac{p_2}{T_2}$$

Or, $\frac{T_3}{T_2} = \frac{P_3}{P_2} = \beta$

Where β is known as pressure or explosion ratio.

Or, $T_2 = \frac{T_3}{\beta} \quad \ldots\ldots\ldots\ldots\ldots\ldots (iii)$

During adiabatic expansion process,

$$\frac{T_4}{T_5}=\left(\frac{v_5}{v_4}\right)^{\gamma-1}=\left(\frac{r}{\rho}\right)^{\gamma-1}\left(\because \frac{v_5}{v_4}=\frac{v_1}{v_4}=\frac{v_1}{v_2}\,X\,\frac{v_2}{v_4}=\frac{v_1}{v_2}\,X\,\frac{v_3}{v_4}=\frac{r}{\rho},\rho,beingthecut-offratio\right).....(iv)$$

During constant pressure heating process,

$$\frac{v_3}{T_3}=\frac{v_4}{T_4}\Rightarrow T_4=T_3\frac{v_4}{v_3}=\rho T_3.....(v)$$

Putting the value of T4 in the eqn. (iv), we get

$$\frac{\rho T_3}{T_5}=\left(\frac{r}{\rho}\right)^{\gamma-1}\Rightarrow T_5=\rho T_3\left(\frac{\rho}{r}\right)^{\gamma-1}$$

Putting the value of T2 in eqn. (ii), we get

$$\frac{\frac{T_3}{\beta}}{T_1}=(r)^{\gamma-1}\Rightarrow T_1=\frac{T_3}{\beta}\frac{1}{(r)^{\gamma-1}}$$

Now inserting the values of T1, T2, T4 and T5 in eqn. (i), we get

$$\eta_{dual}=1-\frac{\left[\rho T_3\left(\frac{\rho}{r}\right)^{\gamma-1}-\frac{T_3}{\beta}\frac{1}{(r)^{\gamma-1}}\right]}{\left[\left(T_3-\frac{T_3}{\beta}\right)+\gamma(\rho T_3-T_3)\right]}=1-\frac{\frac{1}{(r)^{\gamma-1}}(\rho^{\gamma}-\frac{1}{\beta})}{(1-\frac{1}{\beta})+\gamma(\rho-1)}\Rightarrow\eta_{dual}=1-\frac{1}{(r)^{\gamma-1}}\frac{(\beta\rho^{\gamma}-1)}{\left[(\beta-1)+\beta\gamma(\rho-1)\right]}$$

Work done is given by,

$$W=p_3(v_4-v_3)+\frac{p_4v_4-p_5v_5}{\gamma-1}-\frac{p_2v_2-p_1v_1}{\gamma-1}=p_3v_3(\rho-1)+\frac{(p_4v_4-p_5rv_3)-(p_2v_3-p_1rv_3)}{\gamma-1}$$

$$=\frac{p_3v_3(\rho-1)(\gamma-1)+p_4v_3\left(\rho-\frac{p_5}{p_4}r\right)-p_2v_3\left(1-\frac{p_1}{p_2}r\right)}{\gamma-1}$$

Also, $\frac{p_5}{p_4}=\left(\frac{v_4}{v_5}\right)^{\gamma}=\left(\frac{\rho}{r}\right)^{\gamma}$ and $\frac{p_2}{p_1}=\left(\frac{v_1}{v_2}\right)^{\gamma}=r^{\gamma}$

also, $p_3 = p_4, v_2 = v_3, v_5 = v_1$

$$\therefore W = \frac{v_3\left[p_3(\rho-1)(\gamma-1) + p_3(\rho - \rho^{\gamma} r^{1-\gamma}) - p_2(1-r^{1-\gamma})\right]}{(\gamma-1)}$$

$$= \frac{p_2 v_2\left[\beta(\rho-1)(\gamma-1) + \beta(\rho - \rho^{\gamma} r^{1-\gamma}) - (1-r^{1-\gamma})\right]}{\gamma-1}$$

$$= \frac{p_1 (r)^{\gamma} v_1 / r\left[\beta\gamma(\rho-1) + (\beta-1) - r^{1-\gamma}(\beta\rho^{\gamma} - 1)\right]}{\gamma-1}$$

$$= \frac{p_1 v_1 r^{\gamma-1}\left[\beta\gamma(\rho-1) + (\beta-1) - r^{\gamma-1}(\beta\rho^{\gamma} - 1)\right]}{\gamma-1}$$

Mean effective pressure (pm) is given by,

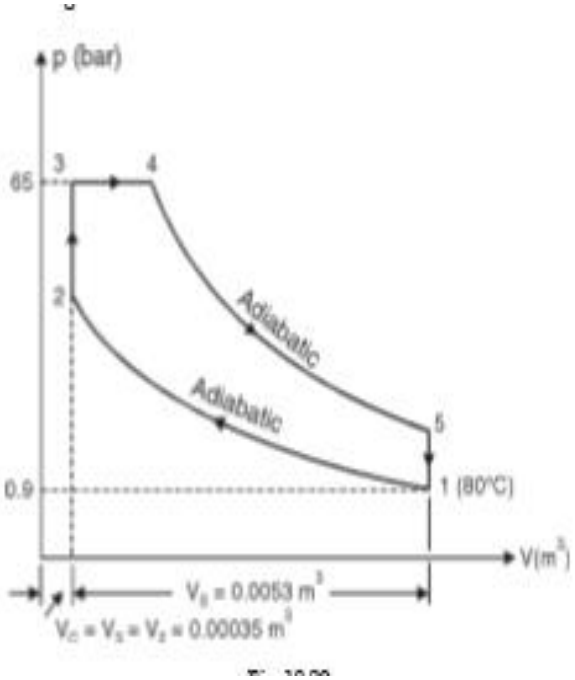
$$p_m = \frac{W}{v_1 - v_2} = \frac{W}{v_1\left(\frac{r-1}{r}\right)} = \frac{p_1 v_1\left[r^{1-\gamma}\beta\gamma(\rho-1) + (\beta-1) - r^{1-\gamma}(\beta\rho^{\gamma} - 1)\right]}{(\gamma-1)v_1\left(\frac{r-1}{r}\right)}$$

$$p_m = \frac{p_1 (r)^{\gamma}\left[\beta(\rho-1) + (\beta-1) - r^{1-\gamma}(\beta\rho^{\gamma} - 1)\right]}{(\gamma-1)(\rho-1)}$$

Example

The swept volume of a diesel engine working on dual cycle is 0.0053 m3 and clearance volume is 0.00035 m3. The maximum pressure is 65 bar. Fuel injection ends at 5 per cent of the stroke. The temperature and pressure at the start of the compression are 80°C and 0.9 bar. Determine the air standard efficiency of the cycle. Take γ for air = 1.4.

Solution.



Swept volume, Vs = 0.0053 m3

Clearance volume, Vc = V3 = V2 = 0.00035 m3

Maximum pressure, p3 = p4 = 65 bar

Initial temperature, T1 = 80 + 273 = 353 K

Initial pressure, p1 = 0.9 bar

ηdual = ?

The efficiency of a dual combustion cycle is given by

$$\eta_{dual} = 1 - \frac{1}{(r)^{\gamma-1}} \frac{(\beta\rho^{\gamma} - 1)}{[(\beta - 1) + \beta\gamma(\rho - 1)]}$$

Compression ratio, $r = \frac{V_1}{V_2} = \frac{V_S + V_C}{V_C} = \frac{0.0053 + 0.00035}{0.00035} = 16.14$

Cutt-off ratio, $\rho = \frac{V_4}{V_3} = \frac{\frac{5}{100}V_S + V_3}{V_3} = \frac{0.05V_S + V_C}{V_C} = \frac{0.05X\,0.0053 + 0.00035}{0.00035} = 1.757 \approx 1.76$

Also during the compression operation 1-2,

$$P_1V_1^{\gamma} = P_2V_2^{\gamma} \Rightarrow \frac{P_2}{P_1} = \left(\frac{V_1}{V_2}\right)^{\gamma} = (16.14)^{1.4} = 49.14 \Rightarrow P_2 = P_1 X\, 49.14 = 0.9\, X\, 49.14 = 44.22 bar$$

Pressure or explosion ratio, $\beta = \frac{P_3}{P_2} = \frac{65}{44.22} = 1.47$

Putting the value of r, ρ and β in eqn. (i), we get

$$\eta_{dual} = 1 - \frac{1}{(16.14)^{1.4-1}}\left[\frac{1.47\, X\, (1.76)^{1.4} - 1}{(1.47-1) + 1.47\, X\, 1.4(1.76-1)}\right] = 0.6383 = 63.83\%$$

Example .

An oil engine working on the dual combustion cycle has a compression ratio 14 and the explosion ratio obtained from an indicator card is 1.4. If the cut-off occurs at 6 per cent of stroke, find the ideal efficiency. Take γ for air = 1.4.

Solution.

Compression ratio, r = 14

Explosion ratio, β = 1.4

If ρ is the cut-off ratio, then, $\frac{\rho - 1}{r - 1} = \frac{6}{100} \; or \; \frac{\rho - 1}{14 - 1} = 0.06$

∴ ρ = 1.78

Ideal efficiency is given by

$$\eta_{dual} = 1 - \frac{1}{(r)^{\gamma-1}} \frac{(\beta\rho^{\gamma} - 1)}{[(\beta - 1) + \beta\gamma(\rho - 1)]} = 1 - \frac{1}{(14)^{1.4-1}}\left[\frac{1.4\, X\, (1.78)^{1.4} - 1}{(1.4-1) + 1.4\, X\, 1.4(1.78-1)}\right] = 0.614 = 61.4\%$$

COMPARISON OF OTTO, DIESEL AND DUAL COMBUSTION CYCLES

Following are the important variable factors which are used as a basis for comparison of the cycles :

- Compression ratio.

- Maximum pressure
- Heat supplied
- Heat rejected
- Net work

Some of the above mentioned variables are fixed when the performance of Otto, Diesel and dual combustion cycles is to be compared.

Efficiency Versus Compression Ratio

- Fig. shows the comparison for the air standard efficiencies of the Otto, Diesel and Dual combustion cycles at various compression ratios and with given cut-off ratio for the Diesel and Dual combustion cycles.
- It is evident from the Fig. that the air standard efficiencies increase with the increase in the compression ratio.
- For a given compression ratio Otto cycle is the most efficient while the Diesel cycle is the least efficient. ($\eta_{otto} > \eta_{dual} > \eta_{diesel}$).

Note. The maximum compression ratio for the petrol engine is limited by detonation. In their respective ratio ranges, the Diesel cycle is more efficient than the Otto cycle.

For the Same Compression Ratio and the Same Heat Input

- A comparison of the cycles (Otto, Diesel and Dual) on the p-v and T-s diagrams for the same compression ratio and heat supplied is shown in the Fig.

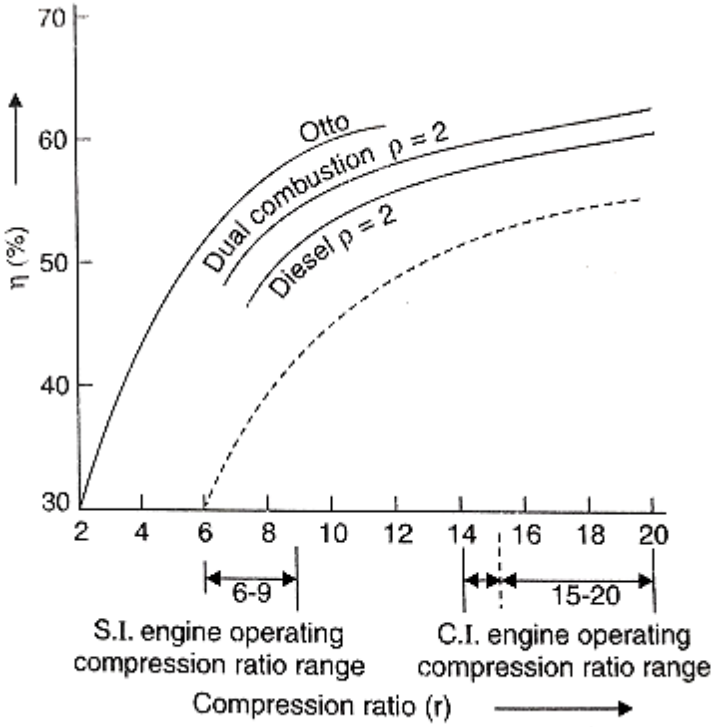


Fig. Comparison of efficiency at various compression ratios.

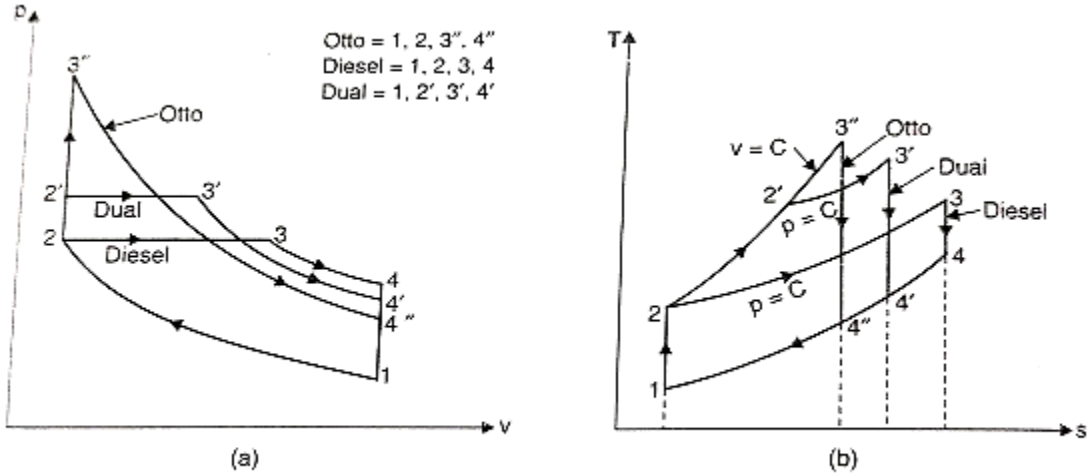


Fig. (*a*) *p-v* diagram, (*b*) *T-s* diagram.

- We know that, $\eta = 1 - \frac{heat rejected}{heat\ \sup\ plied}$ ……(i)

- Since all the cycles reject their heat at the same specific volume, process line from state 4 to 1, the quantity of heat rejected from each cycle is represented by the appropriate area under the line 4 to 1 on

the T-s diagram. As is evident from the eqn. (i) the cycle which has the least heat rejected will have the highest efficiency. Thus, Otto cycle is the most efficient and Diesel cycle is the least efficient of the three cycles. i.e., $\eta_{otto} > \eta_{dual} > \eta_{diesel}$

For Constant Maximum Pressure and Heat Supplied

- Fig. shows the Otto and Diesel cycles on p-v and T-s diagrams for constant maximum pressure and heat input respectively.

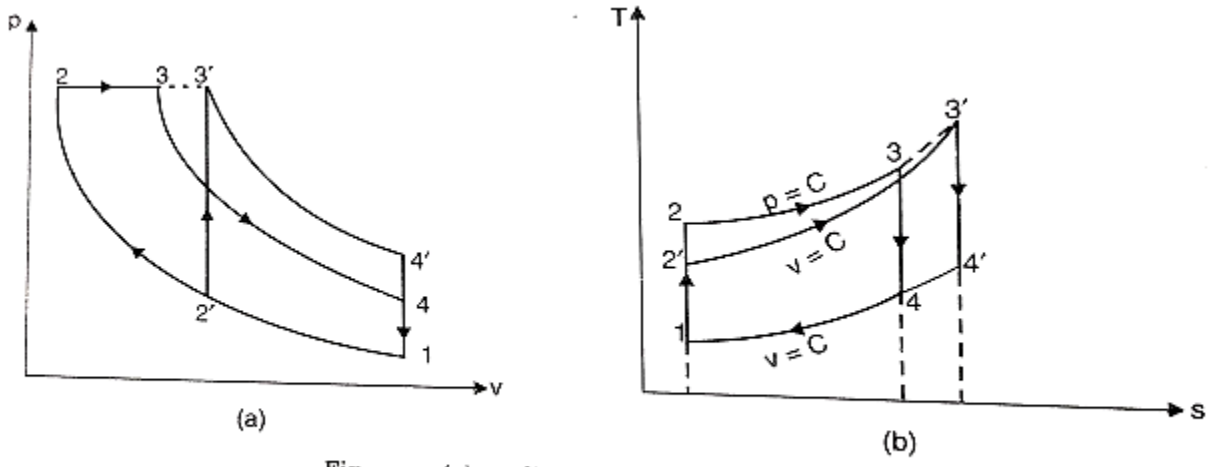


Fig. (a) p-v diagram, (b) T-s diagram.

- For the maximum pressure the points 3 and 3′ must lie on a constant pressure line.
- On T-s diagram the heat rejected from the Diesel cycle is represented by the area under the line 4 to 1 and this area is less than the Otto cycle area under the curve 4′ to 1 ; hence the Diesel cycle is more efficient than the Otto cycle for the condition of maximum pressure and heat supplied.

GAS TURBINE CYCLE—BRAYTON CYCLE

Ideal Brayton Cycle

- Brayton cycle is a constant pressure cycle for a perfect gas. It is also called Joule cycle. The heat transfers are achieved in reversible constant pressure heat exchangers. An ideal gas turbine plant would perform the processes that make up a Brayton cycle.
- The cycle is shown in the Fig.(a) and it is represented on p-v and T-s diagrams as shown in Figs. (b) and (c).

The various operations are as follows :

Operation 1-2. The air is compressed isentropically from the lower pressure p1 to the upper pressure p2, the temperature rising from T1 to T2. No heat flow occurs.

Operation 2-3. Heat flows into the system increasing the volume from V2 to V3 and temperature from T2 to T3 whilst the pressure remains constant at p2. Heat received = mcp (T3 – T2).

Operation 3-4. The air is expanded isentropically from p2 to p1, the temperature falling from T3 to T4. No heat flow occurs.

Operation 4-1. Heat is rejected from the system as the volume decreases from V4 to V1 and the temperature from T4 to T1 whilst the pressure remains constant at p1. Heat rejected = mcp (T4 – T1).

$$\eta_{air-standard} = \frac{Workdone}{heatreceived} = \frac{Heatreceived/cycle - Heatrejected/cycle}{Heatreceived/cycle} = \frac{mc_p(T_3 - T_2) - mc_p(T_4 - T_1)}{mc_p(T_3 - T_2)}$$

$$= 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

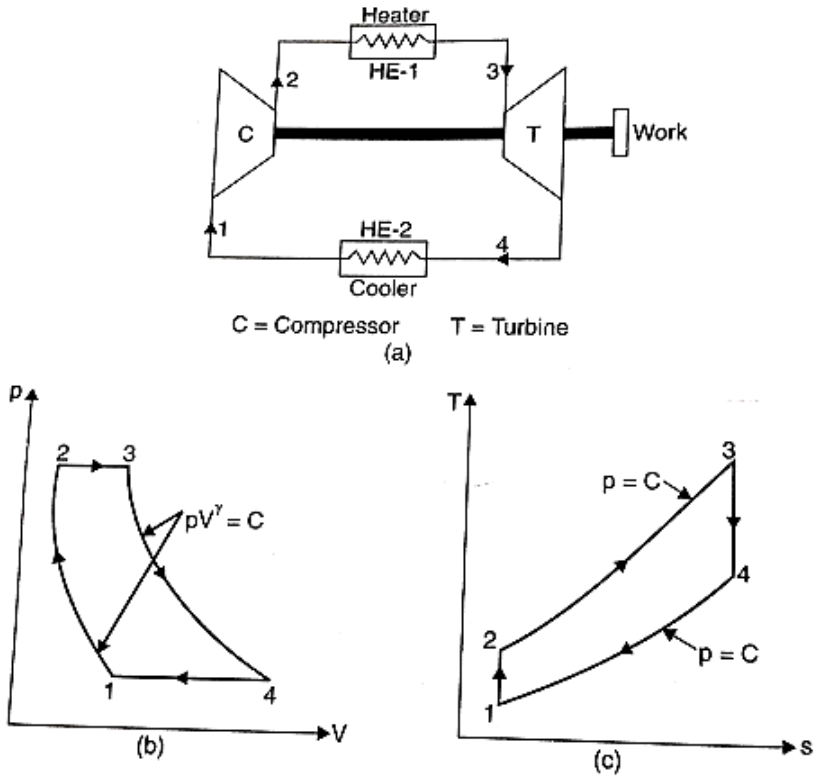


Fig. Brayton cycle : (*a*) Basic components of a gas turbine power plant (*b*) *p-V* diagram (*c*) *T-s* diagram.

Now, from isentropic expansion,

$$\frac{T_2}{T_1}=\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2=T_1(r_p)^{\frac{\gamma-1}{\gamma}}$$, where r_p=pressure ratio

Similarly, $$\frac{T_3}{T_4}=\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_3=T_4(r_p)^{\frac{\gamma-1}{\gamma}}$$

$$\therefore \eta_{air-standard}=1-\frac{T_4-T_1}{T_4(r_p)^{\frac{\gamma-1}{\gamma}}-T_1(r_p)^{\frac{\gamma-1}{\gamma}}}=1-\frac{1}{(r_p)^{\frac{\gamma-1}{\lambda}}}$$

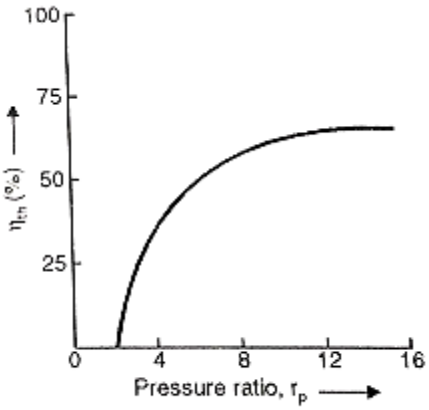


Fig. Effect of pressure ratio on the efficiency of Brayton cycle.

- The eqn. above shows that the efficiency of the ideal joule cycle increases with the pressure ratio. The absolute limit of upper pressure is determined by the limiting temperature of the material of the turbine at the point at which this temperature is reached by the compression process alone, no further heating of the gas in the combustion chamber would be permissible and the work of expansion would ideally just balance the work of compression so that no excess work would be available for external use.

Pressure Ratio for Maximum Work

Here we shall prove that the pressure ratio for maximum work is a function of the limiting temperature ratio.

Work output during the cycle

$$= Heatreceived/cycle - heatrejected/cycle$$

$$= mc_p(T_3 - T_2) - mc_p(T_4 - T_1) = mc_p(T_3 - T_4) - mc_p(T_2 - T_1) = mc_pT_3\left(1 - \frac{T_4}{T_3}\right) - T_1\left(\frac{T_2}{T_1} - 1\right)$$

- In case of a given turbine the minimum temperature T1 and the maximum temperature T3 are prescribed, T1 being the temperature of the atmosphere and T3 the maximum temperature which the metals of turbine would withstand. Consider the specific heat at constant pressure cp to be constant. Then,
- Since, $\frac{T_3}{T_4} = (r_p)^{\frac{\gamma-1}{\gamma}} = \frac{T_2}{T_1}$
- Using the constant, $z = \frac{\gamma - 1}{\gamma}$
- we have, work output/cycle

$$W = K\left[T_3\left(1 - \frac{1}{r_p^z}\right) - T_1(r_p^z - 1)\right]$$

Differentiating with respect to r_p

$$\frac{dW}{dr_p} = K\left[T_3X\frac{z}{r_p(z+1)} - T_1zr_p^{z-1}\right] = 0\ for a \max imum$$

$$\therefore \frac{zT_3}{r_p^{(z+1)}} = T_1z(r_p)^{(z-1)} \Rightarrow r_p^{2z} = \frac{T_3}{T_1} \Rightarrow r_p = (T_3/T_1)^{1/2z} \Rightarrow r_p = (T_3/T_1)^{\frac{\gamma}{2(\gamma-1)}}$$

Thus, the pressure ratio for maximum work is a function of the limiting temperature ratio.

Work Ratio

Work ratio is defined as the ratio of net work output to the work done by the turbine.

$$\therefore workratio = \frac{W_T - W_C}{W_T}$$ [Where, W_T =Work obtained from this turbine ,and W_C=work supplied to compressor.]

$$= \frac{mc_p(T_3 - T_4) - mc_p(T_2 - T_1)}{mc_p(T_3 - T_4)} = 1 - \frac{T_2 - T_1}{T_3 - T_4} = 1 - \frac{T_1}{T_3}\left[\frac{(r_p)^{\frac{\gamma-1}{\gamma}} - 1}{1 - \frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}}\right] = 1 - \frac{T_1}{T_3}(r_p)^{\frac{\gamma-1}{\gamma}}$$

Open Cycle Gas Turbine—Actual Brayton Cycle

- The fundamental gas turbine unit is one operating on the open cycle in which a rotary compressor and a turbine are mounted on a common shaft. Air is drawn into the compressor and after compression passes to a combustion chamber. Energy is supplied in the combustion chamber by spraying fuel into the air stream, and the resulting hot gases expand through the turbine to the atmosphere.

- In order to achieve net work output from the unit, the turbine must develop more gross work output than is required to drive the compressor and to overcome mechanical losses in the drive. The products of combustion coming out from the turbine are exhausted to the atmosphere as they cannot be used any more. The working fluids (air and fuel) must be replaced continuously as they are exhausted into the atmosphere.

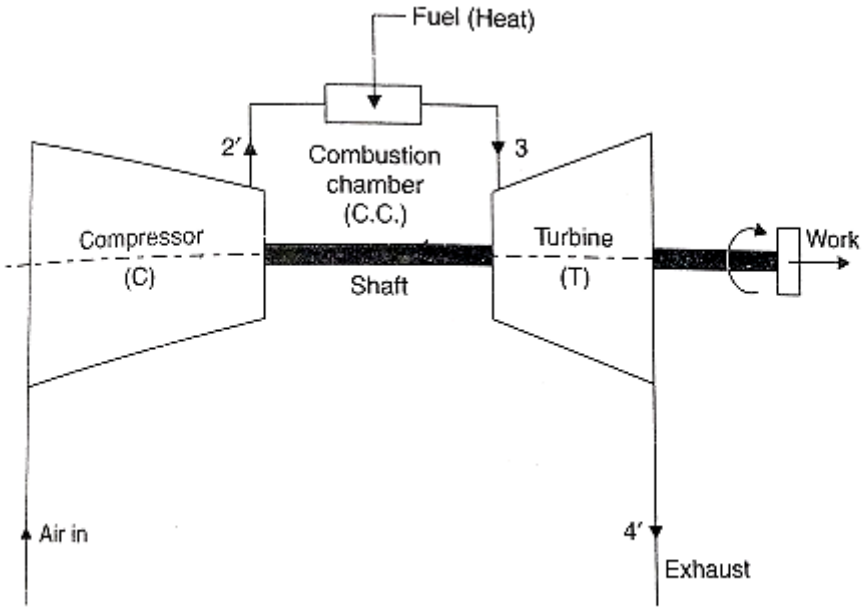


Fig. Open cycle gas turbine.

- If pressure loss in the combustion chamber is neglected, this cycle may be drawn on a T-s diagram
  - 1-2′ represents : irreversible adiabatic compression.
  - 2′-3 represents : constant pressure heat supply in the combustion chamber.

- 3-4′ represents : irreversible adiabatic expansion.
- 1-2 represents : ideal isentropic compression.
- 3-4 represents : ideal isentropic expansion.

➢ Assuming change in kinetic energy between the various points in the cycle to be negligibly small compared with enthalpy changes and then applying the flow equation to each part of cycle, for unit mass, we have

Work input (compressor) = $c_p (T_2' - T_1)$

Heat supplied (combustion chamber) = $c_p (T_3 - T_2')$

Work output (turbine) = $c_p (T_3 - T_4')$

∴ Net work output = Work output – Work input

$= c_p (T_3 - T_4') - c_p(T_2' - T_1)$

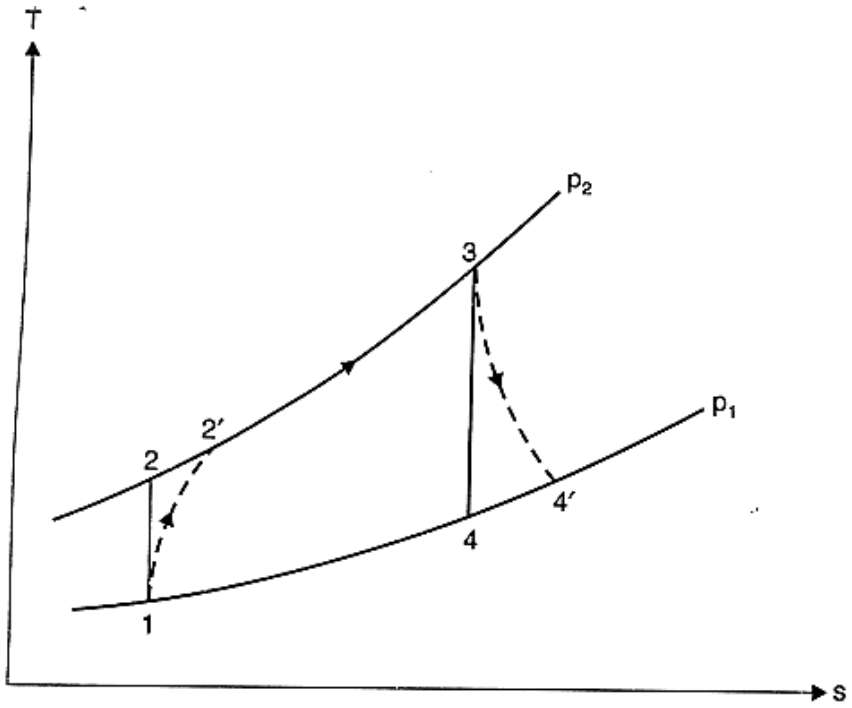


Fig.

And $$\eta_{thermal} = \frac{Networkoutput}{heat\ \sup\ plied} = \frac{c_p(T_3 - T_4') - c_p(T_2' - T_1)}{c_p(T_3 - T_2')}$$

Compressor isentropic efficiency,

$$\eta_{comp} = \frac{Workinputrequiredinisentropiccompression}{Actualworkrequired} = \frac{c_p(T_2 - T_1)}{c_p(T_2' - T_1)} = \frac{T_2 - T_1}{T_2' - T_1}$$

Turbine isentropic efficiency,

$$\eta_{turbine} = \frac{Actualworkoutput}{Isentropicworkoutput} = \frac{c_p(T_3 - T_4')}{c_p(T_3 - T_4)} = \frac{T_3 - T_4'}{T_3 - T_4}$$

Methods for Improvement of Thermal Efficiency of Open Cycle Gas Turbine Plant

The following methods are employed to increase the specific output and thermal efficiency of the plant :

1. Intercooling 2. Reheating 3. Regeneration.

1. Intercooling. A compressor in a gas turbine cycle utilizes the major percentage of power developed by the gas turbine. The work required by the compressor can be reduced by compressing the air in two stages and incorporating an intercooler between the two as shown in Fig.. The corresponding T-s diagram for the unit is shown in Fig.. The actual processes take place as follows :

1-2′ ... L.P. (Low pressure) compression

2′-3 ... Intercooling

3-4′ ... H.P. (High pressure) compression

4′-5 ... C.C. (Combustion chamber)-heating

5-6′ ... T (Turbine)-expansion

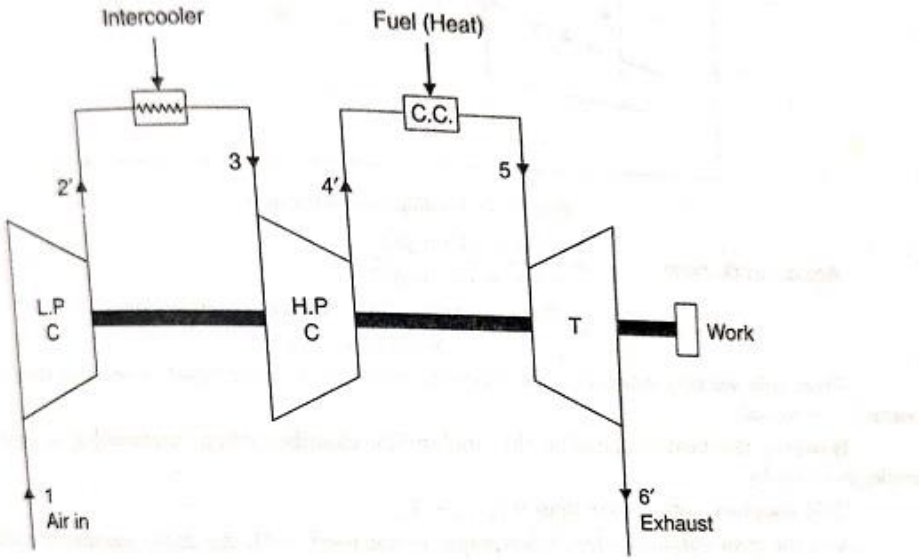


Fig. Turbine plant with intercooler.

The ideal cycle for this arrangement is 1-2-3-4-5-6 ; the compression process without intercooling is shown as 1-L' in the actual case, and 1-L in the ideal isentropic case.

Now, Work input (with intercooling)

$$= c_p(T_2' - T_1) + c_p(T_4' - T_3)$$

Work input (without intercooling)

$$= c_p(T_L' - T_1) = c_p(T_2' - T_1) + c_p(T_L' - T_2')$$

By comparing both equations it can be observed that the work input with intercooling is less than the work input with no intercooling, when cp $(T_4' - T_3)$ is less than $c_p(T_L' - T_2')$.

This is so if it is assumed that isentropic efficiencies of the two compressors, operating separately, are each equal to the isentropic efficiency of the single compressor which would be required if no intercooling were used. Then $(T_4' - T_3) < (T_L' - T_2')$ since the pressure lines diverge on the T-s diagram from left to the right.

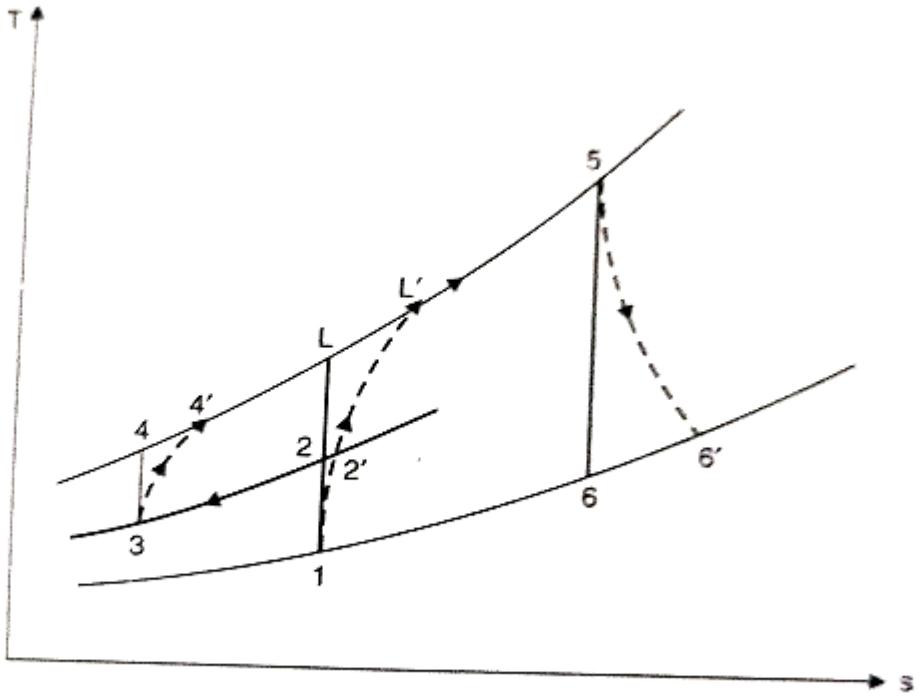


Fig. *T-s* diagram for the unit.

Again, $$workratio = \frac{netwoekoutput}{grossworkoutput} = \frac{workof\ \exp ansion - workofcompression}{workof\ \exp ansion}$$

From this we may conclude that when the compressor work input is reduced then the work ratio is increased.

However the heat supplied in the combustion chamber when intercooling is used in the cycle, is given by,

Heat supplied with intercooling $= c_p(T_5 - T_4')$

Also the heat supplied when intercooling is not used, with the same maximum cycle temperature T5, is given by

Heat supplied without intercooling $= c_p(T_5 - T_L')$

Thus, the heat supplied when intercooling is used is greater than with no intercooling. Although the net work output is increased by intercooling it is found in general that the increase in heat to be supplied causes the thermal efficiency to decrease.

When intercooling is used a supply of cooling water must be readily available. The additional bulk of the unit may offset the advantage to be gained by increasing the work ratio.

2. Reheating. The output of a gas turbine can be amply improved by expanding the gases in two stages with a reheater between the two as shown in Fig.

The H.P. turbine drives the compressor and the L.P. turbine provides the useful power output. The corresponding T-s diagram is shown in Fig.

The line 4'-L' represents the expansion in the L.P. turbine if reheating is not employed.

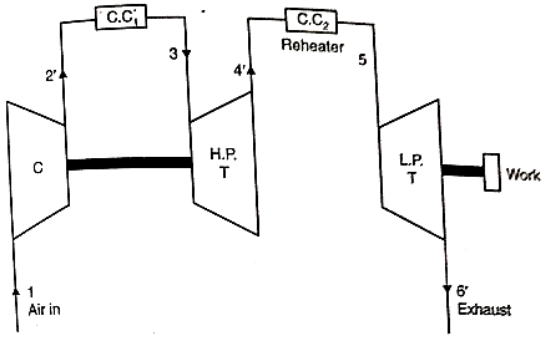


Fig. Gas turbine with reheater.

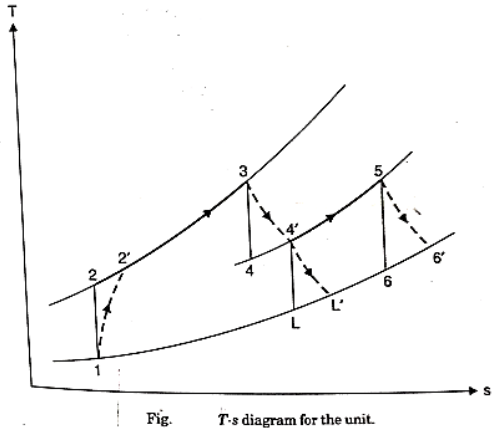


Fig. T-s diagram for the unit.

Neglecting mechanical losses the work output of the H.P. turbine must be exactly equal to the work input required for the compressor i.e., $c_{pa}(T_2' - T_1) = c_{pg}(T_3 - T_4')$

The work output (net output) of L.P. turbine is given by,

Net work output (with reheating) = $c_{pg}(T_5 - T_6')$

and Net work output (without reheating) = $c_{pg}(T_4' - T_L')$

Since the pressure lines diverge to the right on T-s diagram it can be seen that the temperature difference $(T_5 - T_6')$ is always greater than $(T_4' - T_L')$, so that reheating increases the net work output.

Although net work is increased by reheating the heat to be supplied is also increased, and the net effect can be to reduce the thermal efficiency

Heat supplied = $c_{pg}(T_3 - T_2') + c_{pg}(T_5 - T_4')$.

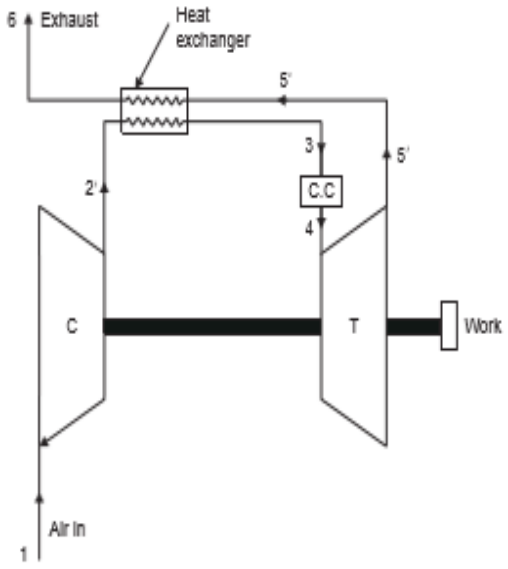
Note. c_{pa} and c_{pg} stand for specific heats of air and gas respectively at constant pressure.

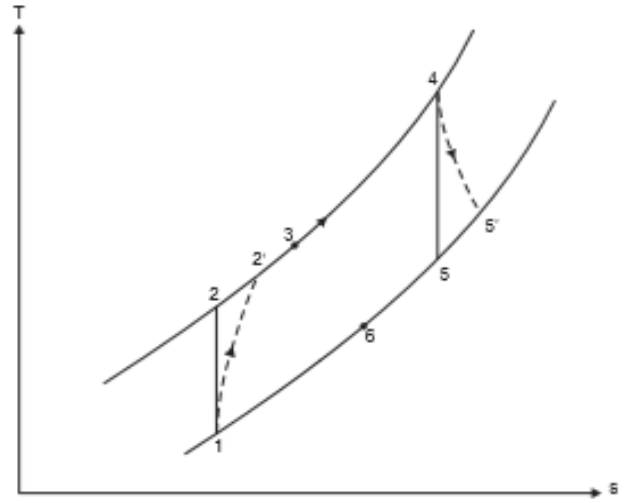
3. Regeneration. The exhaust gases from a gas turbine carry a large quantity of heat with them since their temperature is far above the ambient temperature. They can be used to heat the air coming from the compressor thereby reducing the mass of fuel supplied in the combustion chamber.

Fig. shows a gas turbine plant with a regenerator. The corresponding T-s diagram is shown in Fig.

2'-3 represents the heat flow into the compressed air during its passage through the heat exchanger and 3-4 represents the heat taken in from the combustion of fuel.

Point 6 represents the temperature of exhaust gases at discharge from the heat exchanger. The maximum temperature to which the air could be heated in the heat exchanger is ideally that of exhaust gases, but less than this is obtained in practice because a temperature gradient must exist for an unassisted transfer of energy.





The effectiveness of the heat exchanger is given by :

$$Effectiveness(\varepsilon) = \frac{Increase in enthalpy per kg of air}{Avialable increase in enthalpy per kg of air} = \frac{(T_3 - T_2')}{(T_5' - T_2')}$$

(assuming c_{pa} and c_{pg} to be equal)

A heat exchanger is usually used in large gas turbine units for marine propulsion or industrial power.

Effect of Operating Variables on Thermal Efficiency

The thermal efficiency of actual open cycle depends on the following thermodynamic variables :

(i) Pressure ratio

(ii) Turbine inlet temperature (T3)

(iii) Compressor inlet temperature (T1)

(iv) Efficiency of the turbine ($\eta_{turbine}$)

(v) Efficiency of the compressor (η_{comp}).

Effect of turbine inlet temperature and pressure ratio :

- If the permissible turbine inlet-temperature (with the other variables being constant) of an open cycle gas turbine power plant is increased its thermal efficiency is amply improved. A practical limitation to increasing the turbine inlet temperature, however, is the ability of the material available for the turbine blading to withstand the high rotative and thermal stresses.
- Referring fig. for a given turbine inlet temperature, as the pressure ratio increases, the heat supplied as well as the heat rejected are reduced. But the ratio of change of heat supplied is not the same as the ratio

of change heat rejected. As a consequence, there exists an optimum pressure ratio producing maximum thermal efficiency for a given turbine inlet temperature.

- As the pressure ratio increases, the thermal efficiency also increases until it becomes maximum and then it drops off with a further increase in pressure ratio .Further, as the turbine inlet temperature increases, the peaks of the curves flatten out giving a greater range of ratios of pressure optimum efficiency.

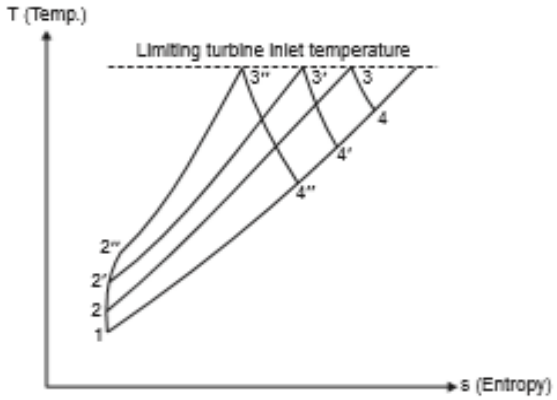
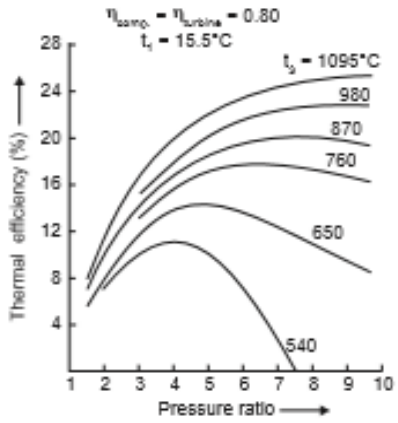


Fig. 13.43

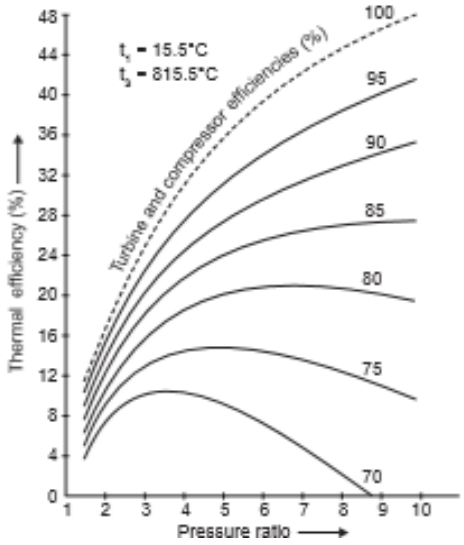


- Following particulars are worthnoting :

Gas temperatures	Efficiency (gas turbine)
550 to 600°C	20 to 22%
900 to 1000°C	32 to 35%
Above 1300°C	more than 50%

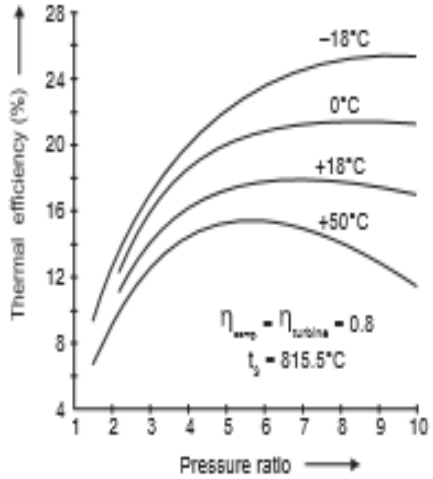
Effect of turbine and compressor efficiencies :

- The thermal efficiency of the actual gas turbine cycle is very sensitive to variations in the efficiencies of the compressor and turbine. There is a particular pressure ratio at which maximum efficiencies occur. For lower efficiencies, the peak of the thermal efficiency occurs at lower pressure ratios and vice versa.



Effect of compressor inlet temperature

- With the decrease in the compressor inlet temperature there is increase in thermal efficiency of the plant. Also the peaks of thermal efficiency occur at high pressure ratios and the curves become flatter giving thermal efficiency over a wider pressure ratio range.



Closed Cycle Gas Turbine (Constant pressure or joule cycle)

- Fig. shows a gas turbine operating on a constant pressure cycle in which the closed system consists of air behaving as an ideal gas. The various operations are as follows :

Operation 1-2 : The air is compressed isentropically from the lower pressure p_1 to the upper pressure p_2, the temperature rising from T_1 to T_2. No heat flow occurs.

Operation 2-3 : Heat flow into the system increasing the volume from V_2 to V_3 and temperature from T_2 to T_3 whilst the pressure remains constant at p_2. Heat received = $mc_p (T_3 – T_2)$.

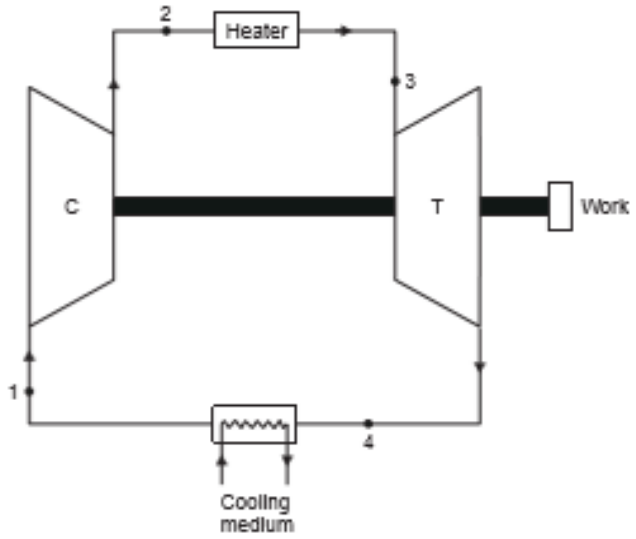
Operation 3-4 : The air is expanded isentropically from p_2 to p_1, the temperature falling from T_3 to T_4. No heat flow occurs.

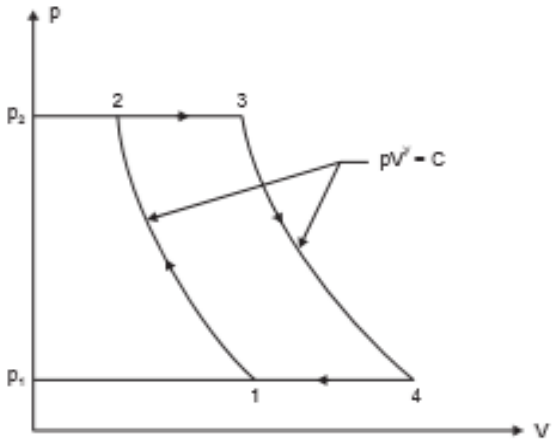
Operation 4-1 : Heat is rejected from the system as the volume decreases from V_4 to V_1 and the temperature from T_4 to T_1 whilst the pressure remains constant at p_1. Heat rejected = $mc_p (T_4 – T_1)$

$$\eta_{air-standard} = \frac{workdone}{heatreceived} = \frac{heatreceived/cycle - heatrejected/cycle}{heatreceived/cycle} = \frac{mc_p(T_3 - T_2) - mc_p(T_4 - T_1)}{mc_p(T_3 - T_2)}$$

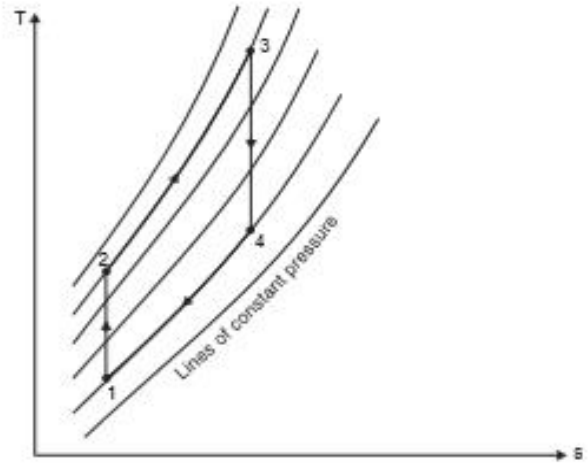
$$= 1 - \frac{T_4 - T_1}{T_3 - T_2}$$

Now, from isentropic expansion, $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_2 = T_1 (r_p)^{\frac{\gamma-1}{\gamma}}$ where r_p =pressure ratio





Similarly, $$\frac{T_3}{T_4}=\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} \Rightarrow T_3 = T_4(r_p)^{\frac{\gamma-1}{\gamma}}$$



$$\therefore \eta_{air-standard} = 1-\frac{T_4-T_1}{T_4(r_p)^{\frac{\gamma-1}{\gamma}}-T_1(r_p)^{\frac{\gamma-1}{\gamma}}} = 1-\frac{1}{(r_p)^{\frac{\gamma-1}{\gamma}}}$$

- The expression shows that the efficiency of the ideal joule cycle increases with the pressure ratio. The absolute limit of pressure is determined by the limiting temperature of the material of the turbine at the point at which this temperature is reached by the compression process alone, no further heating of the gas in the combustion chamber would be permissible and the work of expansion would ideally just balance the work of compression so that no excess work would be available for external use.

- Now we shall prove that the pressure ratio for maximum work is a function of the limiting temperature ratio.

  Work output during the cycle = Heat received/cycle – Heat rejected/cycle

  = $mc_p (T_3 – T_2) – mc_p (T_4 – T_1) = mc_p (T_3 – T_4) – mc_p (T_2 – T_1)$

$$= mc_p T_3(1-\frac{T_4}{T_3}) - T_1\left(\frac{T_2}{T_1}-1\right)$$

- In case of a given turbine the minimum temperature T1 and the maximum temperature T3 are prescribed, T1 being the temperature of the atmosphere and T3 the maximum temperature which the metals of turbine would withstand. Consider the specific heat at constant pressure cp to be constant. Then

  Since, $\frac{T_3}{T_4} = (r_p)^{\frac{\gamma-1}{\gamma}} = \frac{T_2}{T_1}$

  Using the constant $z = \frac{\gamma-1}{\gamma}$

  we have, work output/cycle, $W = K[T_3(1-\frac{1}{r_p^{\,z}}) - T_1(r_p^{\,z}-1)]$

Differentiating with respect to r_p,

$$\frac{dW}{dr_p} = K[T_3 X \frac{z}{r_p(z+1)} - T_1 z r_p^{(z-1)}] = 0$$ for a maximum.

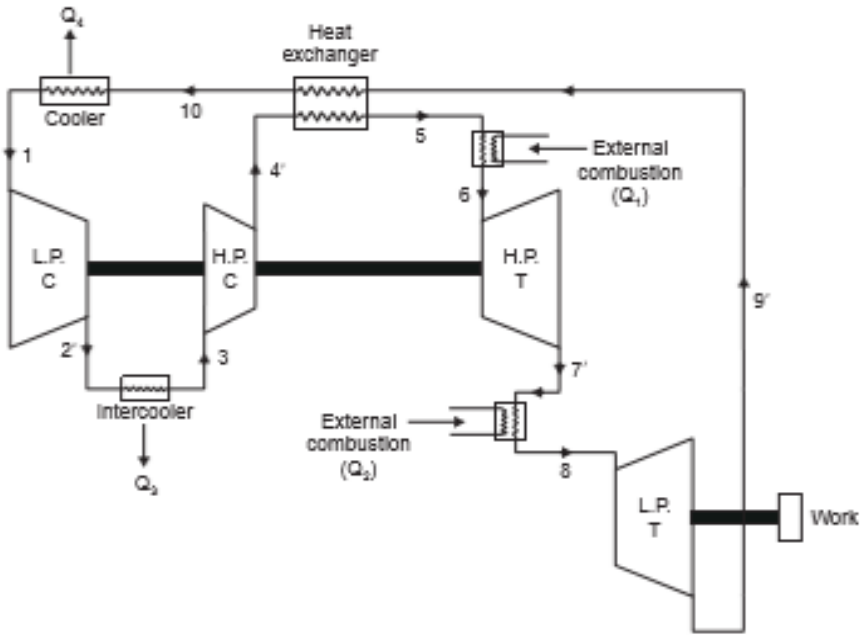
$$\therefore \frac{zT_3}{r_p^{(z+1)}} = T_1 z(r_p^{(z-1)} \Rightarrow r_p^{2z} = \frac{T_3}{T_1}$$

$$\therefore r_p = (T_3/T_1)^{\frac{1}{2z}} \Rightarrow r_p = (T_3/T_1)^{\frac{\gamma}{2(\gamma-1)}}$$

Thus the pressure ratio for maximum work is a function of the limiting temperature ratio.

Fig. shows an arrangement of closed cycle stationary gas turbine plant in which air is continuously circulated. This ensures that the air is not polluted by the addition of combustion waste product, since the heating of air is carried out in the form of heat exchanger shown in the diagram as air heater. The air exhausted from the power turbine is cooled before readmission to L.P. compressor. The various operations as indicated on T-s diagram (Fig. 13.51) are as follows :

Operation 1-2' : Air is compressed from p1 to px in the L.P. compressor.



Operation 2'-3 : Air is cooled in the intercooler at constant pressure px.

Operation 3-4' : Air is compressed in the H.P. compressor from px to p2.

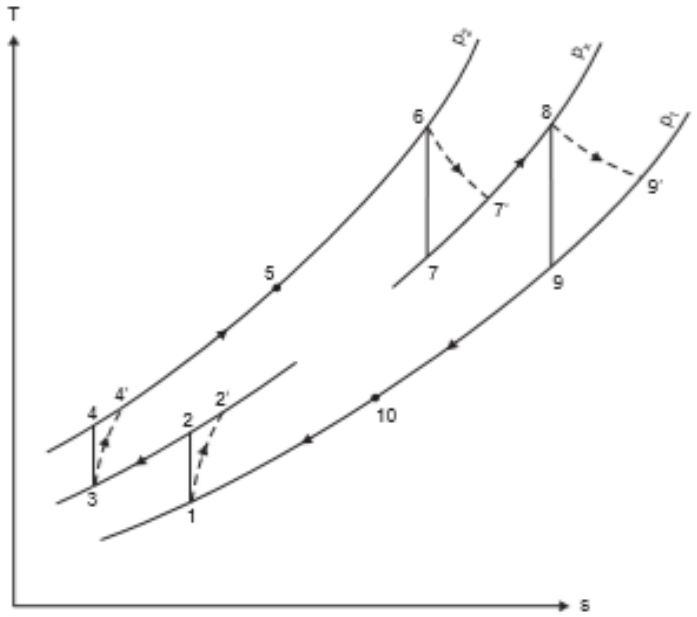
Operation 4'-5 : High pressure air is heated at constant pressure by exhaust gases from power turbine in the heat exchanger to T5.

Operation 5-6 : High pressure air further heated at constant pressure to the maximum temperature T6 by an air heater (through external combustion).

Operation 6-7' : The air is expanded in the H.P. turbine from p2 to px producing work to drive the compressor.
Operation 7'-8 : Exhaust air from the H.P. turbine is heated at constant pressure in the air heater (through external combustion) to the maximum temperature T8(= T6).

Operation 8-9' : The air is expanded in the L.P. turbine from px to p1, producing energy for a flow of work externally.

Operation 9'-10 : Air from L.P. turbine is passed to the heat exchanger where energy is transferred to the air delivered from the H.P. compressor. The temperature of air leaving the heat exchanger and entering the cooler is T10.



Operation 10-11 : Air cooled to T1 by the cooler before entering the L.P. compressor.

The energy balance for the whole plant is as follows :

$$Q_1 + Q_2 - Q_3 - Q_4 = W$$

In a closed cycle plant, in practice, the control of power output is achieved by varying the mass flow by the use of a reservoir in the circuit. The reservoir maintains the design pressure and temperature and therefore achieves an approximately constant level of efficiency for varying loads. In this cycle since it is closed, gases other than air with favourable properties can be used ; furthermore it is possible to burn solid fuels in the combustion heaters. The major factor responsible for inefficiency in this cycle is the large irreversible temperature drop which occurs in the air heaters between the furnace and circulating gas.

Example. Air enters the compressor of a gas turbine plant operating on Brayton cycle at 101.325 kPa, 27°C. The pressure ratio in the cycle is 6. Calculate the maximum temperature in the cycle and the cycle efficiency. Assume $W_T = 2.5\ W_C$, where W_T and W_C are the turbine and the compressor work respectively. Take γ = 1.4.

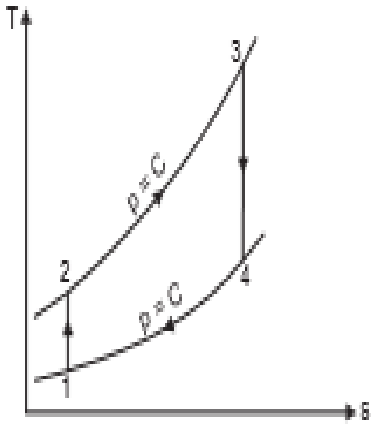
Solution.

Pressure of intake air, p_1 = 101.325 kPa

Temperature of intake air, T_1 = 27 + 273 = 300 K

The pressure ratio in the cycle, r_p = 6

(i) Maximum temperature in the cycle, T_3 :



$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = (r_p)^{\frac{\gamma-1}{\gamma}} = (6)^{\frac{1.4-1}{1.4}} = 1.668$$

$$\Rightarrow T_2 = 1.668 T_1 = 1.668\, X\, 300 = 500.4K$$

$$Also\ \frac{T_3}{T_4} = (r_P)^{\frac{\gamma-1}{\gamma}} = 1.668 \Rightarrow T_4 = \frac{T_3}{1.668}$$

$$But W_T = 2.5 W_C \Rightarrow mc_p(T_3 - T_4) = 2.5 mc_p(T_2 - T_1) \Rightarrow T_3 - \frac{T_3}{1.668} = 2.5(500.4 - 300) = 501$$

$$\Rightarrow T_3 = 1251K = 978^0 C$$

(ii) Cycle efficiency, η_{cycle} :

$$Now T_4 = \frac{T_3}{1.668} = \frac{1251}{1.668} = 750K$$

$$\therefore \eta_{cycle} = \frac{Network}{Heatadded} = \frac{mc_p(T_3 - T_4) - mc_p(T_2 - T_1)}{mc_p(T_3 - T_2)} = \frac{(1251 - 750) - (500.4 - 300)}{(1251 - 500.4)} = 0.4 = 40\%$$

Example. A gas turbine is supplied with gas at 5 bar and 1000 K and expands it adiabatically to 1 bar. The mean specific heat at constant pressure and constant volume are 1.0425 kJ/kg K and 0.7662 kJ/kg K respectively.

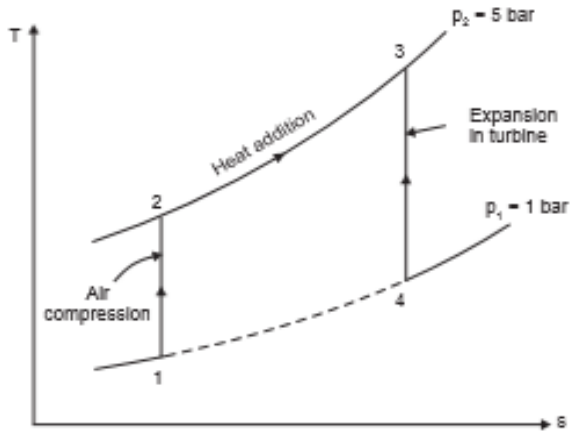
(i) Draw the temperature-entropy diagram to represent the processes of the simple gas turbine system.

(ii) Calculate the power developed in kW per kg of gas per second and the exhaust gas temperature.

Solution. Given : $p_1 = 1$ bar ; $p_2 = 5$ bar ; $T_3 = 1000$ K ; $c_p = 1.0425$ kJ/kg K ; $c_v = 0.7662$ kJ/kg K

$$\gamma = \frac{c_p}{c_v} = \frac{1.0425}{0.7662} = 1.36$$

(i) Temperature-entropy (T-s) diagram :



(ii) Power required :

$$\frac{T_4}{T_3} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{1}{5}\right)^{\frac{1.36-1}{1.36}} = 0.653 \Rightarrow T_4 = 1000\,X\,0.653 = 653K$$

Power developed per kg of gas per second

$= c_p (T_3 - T_4) = 1.0425\ (1000 - 653) = 361.7$ kW.

Example . In a gas turbine plant working on Brayton cycle, the air at inlet is 27°C, 0.1 MPa. The pressure ratio is 6.25 and the maximum temperature is 800°C. The turbine and compressor efficiencies are each 80%. Find compressor work, turbine work, heat supplied, cycle efficiency and turbine exhaust temperature. Mass of air may be considered as 1 kg. Draw T-s diagram.

Solution.

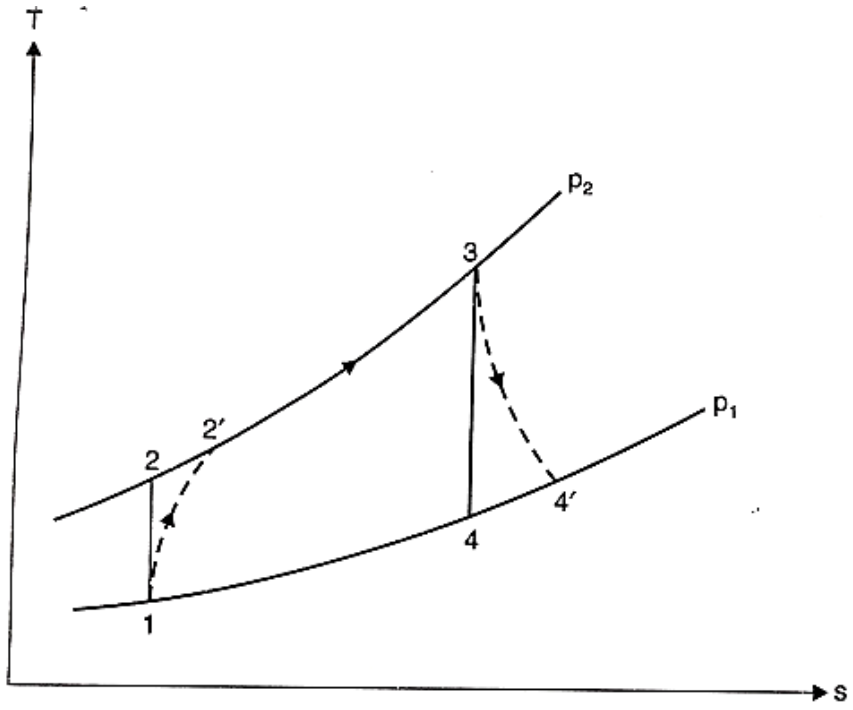


Fig.

Given : $T_1 = 27 + 273 = 300$ K ; $p_1 = 0.1$ MPa ; $r_p = 6.25$, $T_3 = 800 + 273 = 1073$ K ; $\eta_{comp.} = \eta_{turbine} = 0.8$.

For the compression process 1-2, we have

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = (r_p)^{\frac{\gamma-1}{\gamma}} = (6.25)^{\frac{1.4-1}{1.4}} = 1.688$$

$$\Rightarrow T_2 = 1.688 T_1 = 1.668 X\, 300 = 506.4K$$

Also, $$\eta_{comp.} = \frac{T_2 - T_1}{T_2^{'} - T_1} \Rightarrow 0.8 = \frac{506.4 - 300}{T_2^{'} - 300} \Rightarrow T_2^{'} = 558K$$

$$\therefore Cmpressorwork, W_{comp.} = 1xc_p(T_2^{'} - T_1) = 1X1.005X(558 - 300) = 259.29KJ/kg$$

For expansion process 3-4, we have

$$\frac{T_3}{T_4}=\left(\frac{P_3}{P_4}\right)^{\frac{\gamma-1}{\gamma}}=(r_p)^{\frac{\gamma-1}{\gamma}}=(6.25)^{\frac{1.4-1}{1.4}}=1.688\Rightarrow T_4=\frac{T_3}{1.688}=\frac{1073}{1.688}=635.66K$$

$$Also,\eta_{turbine}=\frac{T_3-T_4^{'}}{T_3-T_4}\Rightarrow 0.8=\frac{1073-T_4^{'}}{1073-635.66}\Rightarrow T_4^{'}=723.13K$$

$$\therefore Turbine work, W_{turbine}=1xc_p(T_3-T_4^{'})=1X1.005X(1073-723.13)=351.6KJ/kg$$

$$W_{net}=W_{turbine}-W_{comp.}=351.6-259.29=92.31KJ/kg$$

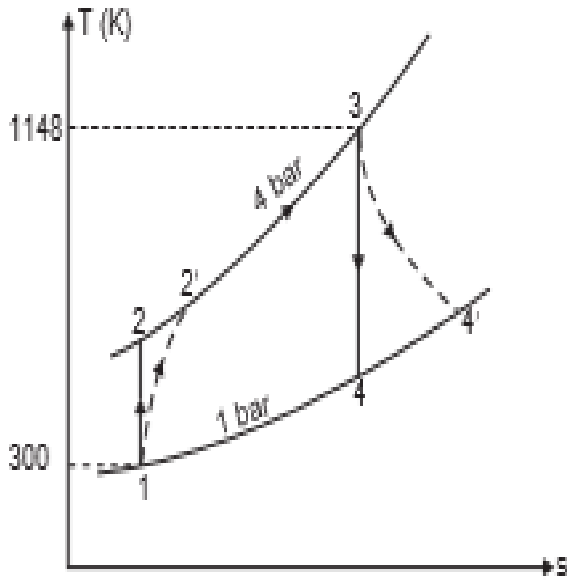
$$Heat\ \sup\ plied, Q_s=1xc_p(T_3-T_2^{'})=1X1.005(1073-558)=517.57KJ/kg$$

$$Cycle efiiciency, \eta_{cycle}=\frac{W_{net}}{Q_s}=\frac{92.31}{517.57}=0.1783=17.83\%$$

$$Turbine exhaust temperature, T_4^{'}=723.13K=450.13^0C$$

Example . Find the required air-fuel ratio in a gas turbine whose turbine and compressor efficiencies are 85% and 80%, respectively. Maximum cycle temperature is 875°C. The working fluid can be taken as air (cp = 1.0 kJ/kg K, γ = 1.4) which enters the compressor at 1 bar and 27°C. The pressure ratio is 4. The fuel used has calorific value of 42000 kJ/kg. There is a loss of 10% of calorific value in the combustion chamber.

Solution. Given : $\eta_{turbine}$ = 85% ; $\eta_{compressor}$ = 80% ; T_3 = 273 + 875 = 1148 K ; T_1 = 27 + 273 = 300 K ; c_p = 1.0 kJ/kg K ; γ = 1.4 ; p_1 = 1 bar; p_2 = 4 bar (since pressure ratio is 4) ; C = 42000 kJ/ kg K, η_{cc} = 90% (since loss in the combustion chamber is 10%)



For isentropic compression 1-2, we have

$$\frac{T_2}{T_1}=\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}}=(r_p)^{\frac{\gamma-1}{\gamma}}=(4)^{\frac{1.4-1}{1.4}}=1.486$$

$$\Rightarrow T_2=1.486T_1=1.486\,X\,300=445.8K$$

$$\eta_{comp.}=\frac{T_2-T_1}{T_2^{'}-T_1}\Rightarrow 0.8=\frac{445.8-300}{T_2^{'}-300}\Rightarrow T_2^{'}=482.2K$$

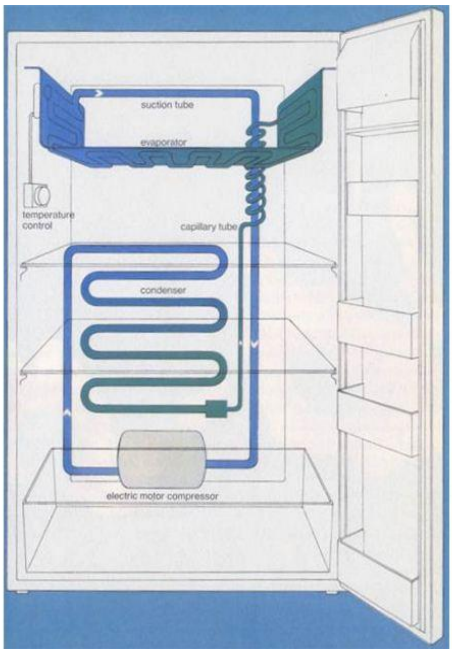
Now, heat supplied by the fuel = heat taken by the burning gases

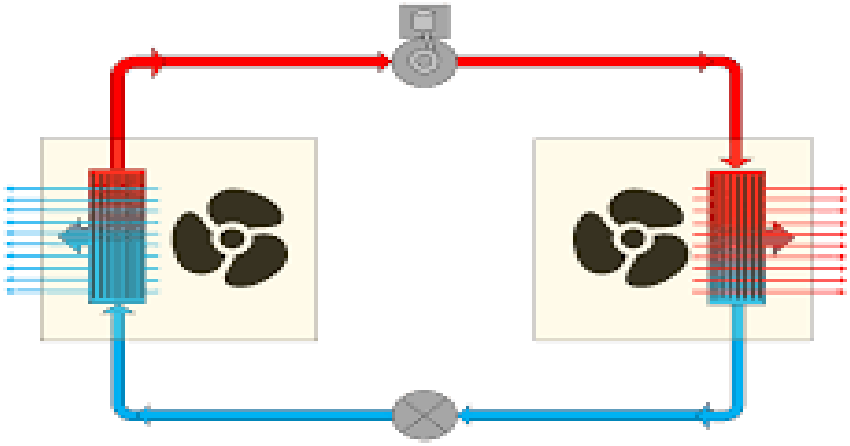
$$0.9Xm_f\,XC=(m_a+m_f)xc_p x(T_3-T_2^{'})\Rightarrow C=\left(\frac{m_a+m_f}{m_f}\right)X\,\frac{c_p(T_3-T_2^{'})}{0.9}=\left(\frac{m_a}{m_f}+1\right)X\,\frac{c_p(T_3-T_2^{'})}{0.9}$$

$$\Rightarrow 42000=\left(\frac{m_a}{m_f}+1\right)X\,\frac{1(1148-482.27)}{0.9}=739.78\left(\frac{m_a}{m_f}+1\right)$$

$$\therefore \frac{m_a}{m_f}=\frac{42000}{739.78}-1=55.77\approx 56$$

$$\therefore A/Fratio=56:1$$





Refrigeration cycles

FUNDAMENTALS OF REFRIGERATION

- Refrigeration is the science of producing and maintaining temperatures below that of the surrounding atmosphere.
- This means the removing of heat from a substance to be cooled. Heat always passes downhill, from a warm body to a cooler one, until both bodies are at the same temperature.
- Maintaining perishables at their required temperatures is done by refrigeration.
- Not only perishables but to-day many human work spaces in offices and factory buildings are airconditioned and a refrigeration unit is the heart of the system.
- Before the advent of mechanical refrigeration water was kept cool by storing it in semiporous jugs so that the water could seep through and evaporate. The evaporation carried away heat and cooled the water. This system was used by the Egyptians and by Indians in the Southwest.
- Natural ice from lakes and rivers was often cut during winter and stored in caves, strawlined pits, and later in sawdust-insulated buildings to be used as required. The Romans carried pack trains of snow from Alps to Rome for cooling the Emperor's drinks. Though these methods of cooling all make use of natural phenomena, they were used to maintain a lower temperature in a space or product and may properly be called refrigeration.

In simple, refrigeration means the cooling of or removal of heat from a system. The equipment employed to maintain the system at a low temperature is termed as refrigerating system and the system which is kept at lower temperature is called refrigerated system.

Refrigeration is generally produced in one of the following three ways :

(i) By melting of a solid.

(ii) By sublimation of a solid.

(iii) By evaporation of a liquid.

Most of the commercial refrigeration is produced by the evaporation of a liquid called refrigerant. Mechanical refrigeration depends upon the evaporation of liquid refrigerant and its circuit includes the equipments naming evaporator, compressor, condenser and expansion valve. It is used for preservation of food, manufacture of ice, solid carbon dioxide and control of air temperature and humidity in the air-conditioning system.

Important refrigeration applications :

1. Ice making
2. Transportation of foods above and below freezing
3. Industrial air-conditioning
4. Comfort air-conditioning
5. Chemical and related industries
6. Medical and surgical aids
7. Processing food products and beverages
8. Oil refining and synthetic rubber manufacturing
9. Manufacturing and treatment of metals
10. Freezing food products
11. Miscellaneous applications :
    - (i) Extremely low temperatures
    - (ii) Plumbing
    - (iii) Building construction etc.

Elements of Refrigeration Systems

All refrigeration systems must include atleast four basic units as given below :

(i) A low temperature thermal "sink" to which heat will flow from the space to be cooled.

(ii) Means of extracting energy from the sink, raising the temperature level of this energy, and delivering it to a heat receiver.

(iii) A receiver to which heat will be transferred from the high temperature high-pressure refrigerant.

(iv) Means of reducing of pressure and temperature of the refrigerant as it returns from the receiver to the "sink".

Refrigeration Systems

The various refrigeration systems may be enumerated as below :

1. Ice refrigeration

2. Air refrigeration system

3. Vapour compression refrigeration system

4. Vapour absorption refrigeration system

5. Special refrigeration systems

(i) Adsorption refrigeration system

(ii) Cascade refrigeration system

(iii) Mixed refrigeration system

(iv) Vortex tube refrigeration system

(v) Thermoelectric refrigeration

(vi) Steam jet refrigeration system.

Co-efficient of Performance (C.O.P.)

The performance of a refrigeration system is expressed by a term known as the ''co-efficient of performance'', which is defined as the ratio of heat absorbed by the refrigerant while passing through the evaporator to the work input required to compress the refrigerant in the compressor ; in short it is the ratio between heat extracted and work done (in heat units).

If, Rn = Net refrigerating effect, W = Work expanded in by the machine during the same interval of time,

Then, $C.O.P. = \frac{R_n}{W}$

And $\text{Re}\,lative C.O.P. = \frac{Actual C.O.P.}{Theoretical C.O.P.}$

where, Actual C.O.P. = Ratio of R_n and W actually measured during a test and,

Theoretical C.O.P. = Ratio of theoretical values of R_n and W obtained by applying laws of thermodynamics to the refrigeration cycle.

Standard Rating of a Refrigeration Machine

- The rating of a refrigeration machine is obtained by refrigerating effect or amount of heat extracted in a given time from a body.
- The rating of the refrigeration machine is given by a unit of refrigeration known as ''standard commercial tonne of refrigeration'' which is defined as the refrigerating effect produced by the melting of 1 tonne of ice from and at 0°C in 24 hours.
- Since the latent heat of fusion of ice is 336 kJ/kg, the refrigerating effect of 336 × 1000 kJ in 24 hours is rated as one tonne, i.e.,

$$\text{1 tonne of refrigeration (TR)} = \frac{336X1000}{24} = 14000KJ/h$$

Note : Ton of refrigeration (TR). A ton of refrigeration is basically an American unit of refrigerating effect (R.E.). It originated from the rate at which heat is required to be removed to freeze one ton of water from and at 0°C. Using American units this is equal to removal of 200 BTU of heat per minute, and MKS unit it is adopted as 50 kcal/min or 3000 kcal/hour. In S.I. units its conversion is rounded of to 3.5 kJ/s (kW) or 210 kJ/min.

AIR REFRIGERATION SYSTEM

Introduction

- Air cycle refrigeration is one of the earliest methods of cooling developed. It became obsolete for several years because of its low co-efficient of performance (C.O.P.) and high operating costs. It has, however, been applied to aircraft refrigeration systems, where with low equipment weight, it can utilise a portion of the cabin air according to the supercharger capacity. The main characteristic feature of air refrigeration system, is that throughout the cycle the refrigerant remains in gaseous state.
- The air refrigeration system can be divided in two systems :

  (i) Closed system

  (ii) Open system.
- In closed (or dense air) system the air refrigerant is contained within the piping or components parts of the system at all times and refrigerator with usually pressures above atmospheric pressure.
- In the open system the refrigerator is replaced by the actual space to be cooled with the air expanded to atmospheric pressure, circulated through the cold room and then compressed to the cooler pressure. The pressure of operation in this system is inherently limited to operation at atmospheric pressure in the refrigerator.
- A closed system claims the following advantages over open system :

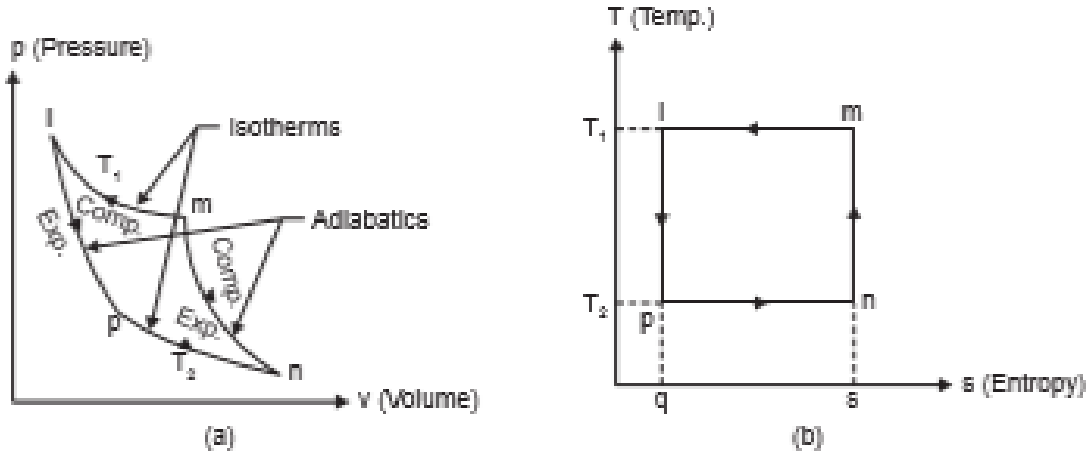
(i) In a closed system the suction to compressor may be at high pressure. The sizes of expander and compressor can be kept within reasonable limits by using dense air ;

(ii) In open air system, the air picks up moisture from the products kept in the refrigerated chamber ; the moisture may freeze during expansion and is likely to choke the valves whereas it does not happen in closed system and

(iii) In open system, the expansion of the refrigerant can be carried only upto atmospheric pressure prevailing in the cold chamber but for a closed system there is no such restriction.

Reversed Carnot Cycle

- If a machine working on reversed Carnot cycle is driven from an external source, it will work or function as a refrigerator. The production of such a machine has not been possible practically because the adiabatic portion of the stroke would need a high speed while during isothermal portion of stroke a very low speed will be necessary. This variation of speed during the stroke, however is not practicable.
- p-V and T-s diagrams of reversed Carnot cycle are shown in Figs.(a) and (b). Starting from point l, the clearance space of the cylinder is full of air, the air is then expanded adiabatically to point p during which its temperature falls from T_1 to T_2, the cylinder is put in contact with a cold body at temperature T_2.
- The air is then expanded isothermally to the point n, as a result of which heat is extracted from the cold body at temperature T_2. Now the cold body is removed ; from n to m air undergoes adiabatic compression with the assistance of some external power and temperature rises to T_1. A hot body at temperature T_1 is put in contact with the cylinder. Finally the air is compressed isothermally during which process heat is rejected to the hot body.



- Refer Fig. (b) Heat abstracted from the cold body = Area 'npqs' = $T_2 \times pn$

  Work done per cycle = Area 'lpnm' = $(T_1 - T_2) \times pn$

- Co-efficient of performance,

$$C.O.P. = \frac{Heatextractedfromthecoldbody}{workdonepercycle} = \frac{Area'npqs'}{Area'lpnm'} = \frac{T_2 Xpn}{(T_1 - T_2)Xpn} = \frac{T_2}{T_1 - T_2}$$

- Since the co-efficient of performance (C.O.P.) means the ratio of the desired effect in kJ/kg to the energy supplied in kJ/kg, therefore C.O.P. in case of Carnot cycle run either as a refrigerating machine or a heat pump or as a heat engine will be as given below :

(i) For a reversed Carnot cycle 'refrigerating machine' :

$$C.O.P._{(ref)} = \frac{\text{Heatextrac tedfromthe coldbody/c ycle}}{workdonepercycle} = \frac{T_2 Xpn}{(T_1 - T_2)Xpn} = \frac{T_2}{T_1 - T_2}$$

(ii) For a Carnot cycle 'heat pump' :

$$C.O.P._{(heatpump)} = \frac{Heatrejectedtothehotbody/cycle}{Workdonepercycle} = \frac{T_1 Xlm}{(T_1 - T_2)Xpn}$$

$$= \frac{T_1 Xpn}{(T_1 - T_2)Xpn}$$

$$= \frac{T_1}{T_1 - T_2}$$

$$= 1 + \frac{T_2}{T_1 - T_2}$$

This indicates that C.O.P. of heat pump is greater than that of a refrigerator working on reversed Carnot cycle between the same temperature limits T_1 and T_2 by unity.

(iii) For a Carnot cycle 'heat engine' :

$$C.O.P._{(heatengine)} = \frac{workobtained/cycle}{Heat\ sup\ plied/cycle} = \frac{(T_1 - T_2)Xpn}{T_1 Xlm} = \frac{(T_1 - T_2)Xpn}{T_1 Xpn} = \frac{T_1 - T_2}{T_1}$$

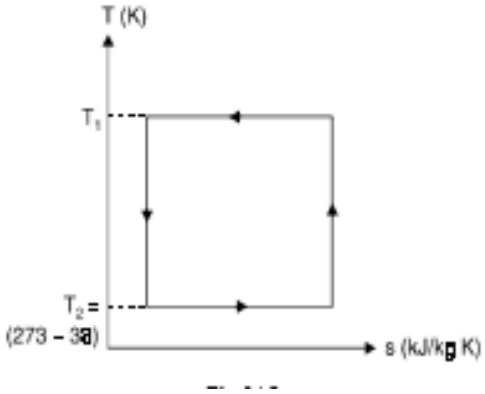
Example. A Carnot refrigerator requires 1.3 kW per tonne of refrigeration to maintain a region at low temperature of – 38°C. Determine : (i) C.O.P. of Carnot refrigerator (ii) Higher temperature of the cycle (iii) The heat delivered and C.O.P. when this device is used as heat pump.

Solution. $T_2 = 273 - 38 = 235$ K

Power required per tonne of refrigeration = 1.3 Kw

(i) C.O.P. of Carnot refrigerator :

$$C.O.P._{CARNOTref} = \frac{Heatabsorbed}{workdone} = \frac{1tonne}{1.3} = \frac{14000kJ/h}{1.3X\,60X\,60kJ/h} = 2.99$$



(ii) Higher temperature of the cycle, T1 :

$$C.O.P._{CARNOTref} = \frac{T_2}{T_1 - T_2} \Rightarrow 2.99 = \frac{235}{T_1 - 235} \Rightarrow T_1 = 313.6K = 40.6^0 C$$

(iii) Heat delivered as heat pump = Heat absorbed + Work done

$$= \frac{14000}{60} + 1.3X\,6 = 311.3kJ/\min$$

$$= C.O.P._{HEATPUMP} = \frac{Heatdelivered}{workdone} = \frac{311.3}{1.3X\,60} = 3.99$$

Example. A refrigerating system operates on the reversed Carnot cycle. The higher temperature of the refrigerant in the system is 35°C and the lower temperature is – 15°C. The capacity is to be 12 tonnes. Neglect all losses. Determine : (i) Co-efficient of performance. (ii) Heat rejected from the system per hour. (iii) Power required.

Solution. (i) T1 = 273 + 35 = 308 K

T2 = 273 – 15 = 258 K

Capacity = 12 tonne

$$C.O.P. = \frac{T_2}{T_1 - T_2} = \frac{258}{308 - 258} = 5.16$$

(ii) Heat rejected from the system per hour :

$$C.O.P. = \frac{\text{Re } frigeratingeffect}{workinput} \Rightarrow 5.16 = \frac{12X14000KJ/h}{workinput} \Rightarrow workinput = \frac{12X14000KJ/h}{5.16} = 32558KJ/h$$

Thus, heat rejected/hour = Refrigerating effect/hour + Work input/hour = 12 × 14000 + 32558 = 200558 kJ/h.

(iii) Power required :

$$Powerrequired = \frac{workinput/hour}{60X60} = \frac{32558}{60X60} = 9.04KW$$

Reversed Brayton Cycle

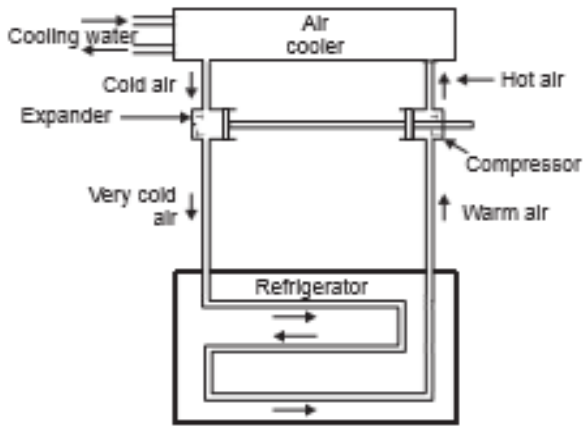
Fig. shows a schematic diagram of an air refrigeration system working on reversed Brayton cycle. Elements of this systems are :

1. Compressor

2. Cooler (Heat exchanger)

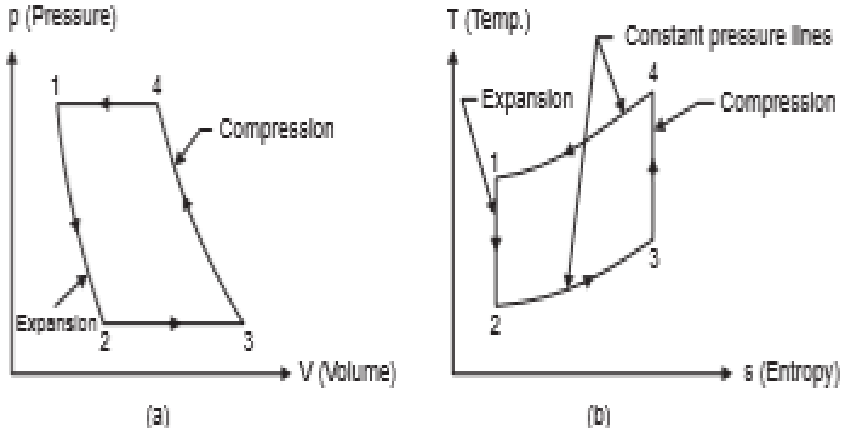
3. Expander

4. Refrigerator.



In this system, work gained from expander is employed for compression of air, consequently less external work is needed for operation of the system. In practice it may or may not be done e.g., in some aircraft refrigeration systems which employ air refrigeration cycle the expansion work may be used for driving other devices.

This system uses reversed Brayton cycle which is described below :



Figs. (a) and (b) shows p-V and T-s diagrams for a reversed Brayton cycle. Here it is assumed that (i) absorption and rejection of heat are constant pressure processes and (ii) Compression and expansion are isentropic processes.

Considering m kg of air :

Heat absorbed in refrigerator, Qadded = $m \times c_p \times (T_3 - T_2)$

Heat rejected is cooler, Qrejected = $m \times c_p \times (T_4 - T_1)$

If the process is considered to be polytropic, the steady flow work of compression is given by,

$$W_{comp} = \frac{n}{n-1}(P_4V_4 - P_3V_3)$$

Similarly work of expansion is given by,

$$W_{comp} = \frac{n}{n-1}(P_1V_1 - P_2V_2)$$

Equations above may easily be reduced to the theoretical isentropic process shown in Fig. (b) by substituting $\gamma = n$ and the known relationship.

$$R = c_p\left(\frac{\gamma - 1}{\gamma}\right)$$

The net external work required for operation of the cycle

= Steady flow work of compression – Steady flow work of expansion

= $W_{comp.} - W_{exp.}$

$$=\left(\frac{n}{n-1}\right)(P_4V_4-P_3V_3-P_1V_1+P_2V_2)=\left(\frac{n}{n-1}\right)mR(T_4-T_3-T_1+T_2)=\left(\frac{n}{n-1}\right)mR(T_4-T_3-T_1+T_2)$$ Bu

t, $R=c_p\left(\frac{\gamma-1}{\gamma}\right)$

(J = 1 in S.I. units)

$$W_{comp}-W_{\exp}=\left(\frac{n}{n-1}\right)\left(\frac{\gamma-1}{\gamma}\right)mc_p(T_4-T_3-T_1+T_2)$$

For isentropic compression and expansion,

$W_{net} = mc_p(T_4 – T_3 + T_2 – T_1)$

Now according to law of conservation of energy the net work on the gas must be equivalent to the net heat rejected.

Now, $C.O.P.=\frac{W_{added}}{Q_{rejected}-Q_{added}}=\frac{Q_{added}}{W_{net}}$

For the air cycle assuming polytropic compression and expansion, co-efficient of performance is :

$$C.O.P.=\frac{mc_p(T_3-T_2)}{\left(\frac{n}{n-1}\right)\left(\frac{\gamma-1}{\gamma}\right)mc_p(T_4-T_3+T_2-T_1)}=\frac{(T_3-T_2)}{\left(\frac{n}{n-1}\right)\left(\frac{\gamma-1}{\gamma}\right)(T_4-T_3+T_2-T_1)}$$

Note.

- The reversed Brayton cycle is same as the Bell-Coleman cycle. Conventionally Bell-Coleman cycle refers to a closed cycle with expansion and compression taking place in reciprocating expander and compressor respectively, and heat rejection and heat absorption taking place in condenser and evaporator respectively.
- With the development of efficient centrifugal compressors and gas turbines, the processes of compression and expansion can be carried out in centrifugal compressors and gas turbines respectively. Thus the shortcoming encountered with conventional reciprocating expander and compressor is overcome. Reversed Brayton cycle finds its application for air-conditioning of aeroplanes where air is used as refrigerant.

Merits and Demerits of Air refrigeration System

Merits

1. Since air is non-flammable, therefore there is no risk of fire as in the machine using NH_3 as the refrigerant.

2. It is cheaper as air is easily available as compared to the other refrigerants.

3. As compared to the other refrigeration systems the weight of air refrigeration system per tonne of refrigeration is quite low, because of this reason this system is employed in aircrafts.

Demerits

1. The C.O.P. of this system is very low in comparison to other systems.

2. The weight of air required to be circulated is more compared with refrigerants used in other systems. This is due to the fact that heat is carried by air in the form of sensible heat.

Example. A Bell-Coleman refrigerator operates between pressure limits of 1 bar and 8 bar. Air is drawn from the cold chamber at 9°C, compressed and then it is cooled to 29°C before entering the expansion cylinder. Expansion and compression follow the law $pv^{1.35}$ = constant. Calculate the theoretical C.O.P. of the system.

For air take $\gamma = 1.4$, $c_p = 1.003$ kJ/kg K.

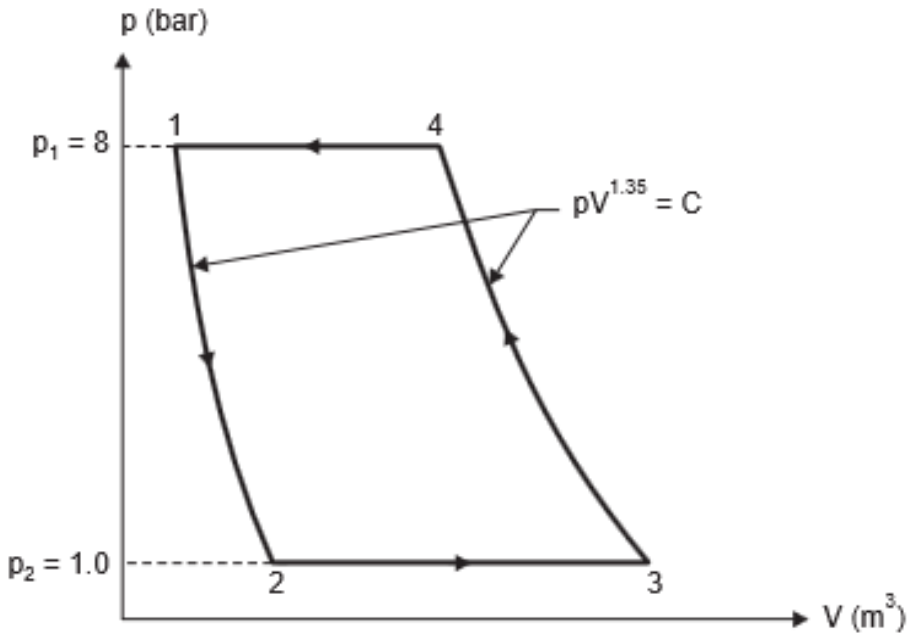
Solution. Fig. shows the working cycle of the refrigerator.

Given : $p_2 = 1.0$ bar ;

$p_1 = 8.0$ bar ;

$T_3 = 9 + 273 = 282$ K ;

$T_4 = 29 + 273 = 302$ K.



Considering polytropic compression 3-4, we have

$$\frac{T_4}{T_3}=\left(\frac{P_1}{P_2}\right)^{\frac{n-1}{n}}=\left(\frac{8}{1}\right)^{\frac{1.35-1}{1.35}}=1.71$$

or $T_4 = T_3 \times 1.71 = 282 \times 1.71 = 482.2$ K

Again, considering polytropic expansion 1-2, we have

$$\frac{T_1}{T_2}=\left(\frac{P_1}{P_2}\right)^{\frac{n-1}{n}}=\left(\frac{8}{1}\right)^{\frac{1.35-1}{1.35}}=1.71\Rightarrow T_2=\frac{T_1}{1.71}=\frac{302}{1.71}=176.6K$$

Heat extracted from cold chamber per kg of air

$= c_p (T3 – T2) = 1.003 (282 – 176.6) = 105.7$ kJ/kg.

Heat rejected in the cooling chamber per kg of air

$= c_p (T4 – T1) = 1.003 (482.2 – 302) = 180.7$ kJ/kg.

Since the compression and expansion are not isentropic, difference between heat rejected and heat absorbed is not equal to the work done because there are heat transfers to the surroundings and from the surroundings during compression and expansion.

To find the work done, the area of the diagram '1-2-3-4' is to be considered :

Work done $$=\frac{n}{n-1}(P_4V_4-P_3V_3)-\frac{n}{n-1}(P_1V_1-P_2V_2)=\frac{n}{n-1}R\left[\left(T_4-T_3\right)-(T_1-T_2)\right]$$

The value of R can be calculated as follows

$$\frac{c_p}{c_v}=\gamma\Rightarrow c_v=\frac{c_p}{\gamma}=\frac{1.003}{1.4}=0.716$$

$R = (c_p – c_v) = 1.003 – 0.716 = 0.287$ kJ/kg K.

∴ Work done $$=\frac{1.35}{0.35}X\,0.287\left[\left(482.2-282\right)-(302-176.6)\right]=82.8KJ/kg$$

$$\therefore C.O.P.=\frac{Heatabstracted}{workdone}=\frac{105.7}{82.4}=1.27$$

SIMPLE VAPOUR COMPRESSION SYSTEM

Introduction

- Out of all refrigeration systems, the vapour compression system is the most important system from the view point of commercial and domestic utility. It is the most practical form of refrigeration. In this system the working fluid is a vapour. It readily evaporates and condenses or changes alternately between the vapour and liquid phases without leaving the refrigerating plant. During evaporation, it absorbs heat from the cold body. This heat is used as its latent heat for converting it from the liquid to vapour. In condensing or cooling or liquifying, it rejects heat to external body, thus creating a cooling effect in the working fluid. This refrigeration system thus acts as a latent heat pump since it pumps its latent heat from the cold body or brine and rejects it or delivers it to the external hot body or cooling medium. The principle upon which the vapour compression system works apply to all the vapours for which tables of Thermodynamic properties are available.

Simple Vapour Compression Cycle

In a simple vapour compression system fundamental processes are completed in one cycle.

These are :

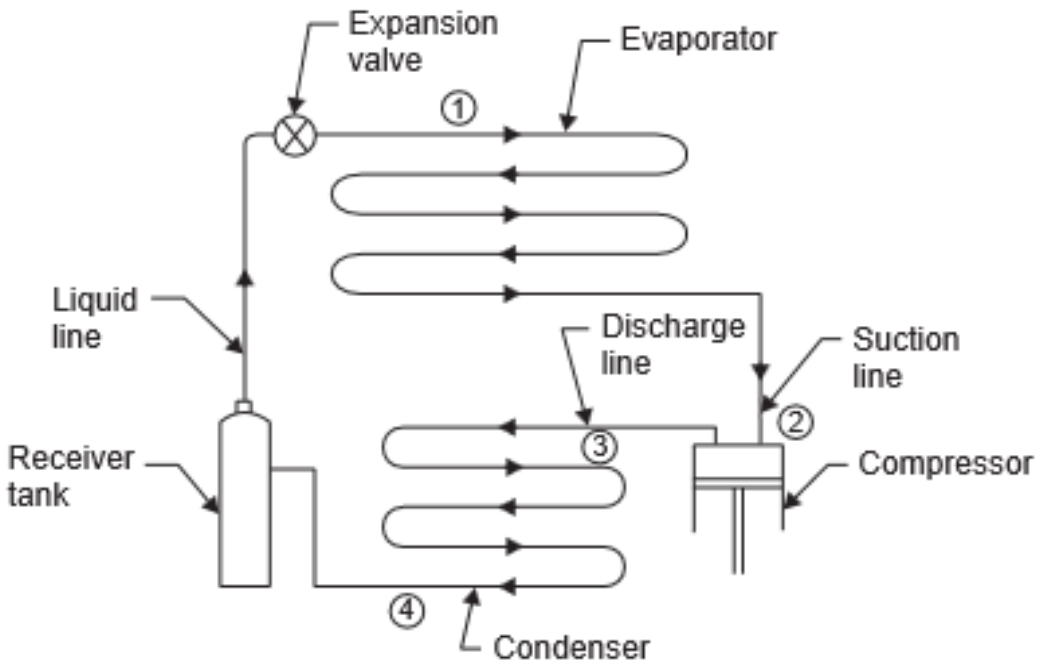
1. Compression

2. Condensation

3. Expansion

4. Vapourisation.

The flow diagram of such a cycle is shown in Fig.



- The vapour at low temperature and pressure (state '2') enters the "compressor" where it is compressed isentropically and subsequently its temperature and pressure increase considerably (state '3'). This vapour after leaving the compressor enters the ''condenser" where it is condensed into high pressure liquid (state '4') and is collected in a "receiver tank". From receiver tank it passes through the "expansion valve", here it is throttled down to a lower pressure and has a low temperature (state '1'). After finding its way through expansion "valve" it finally passes on to "evaporator" where it extracts heat from the surroundings or circulating fluid being refrigerated and vapourises to low pressure vapour (state '2').

Merits and demerits of vapour compression system over Air refrigeration system :

Merits :

1. C.O.P. is quite high as the working of the cycle is very near to that of reversed Carnot cycle.

2. When used on ground level the running cost of vapour-compression refrigeration system is only 1/5th of air refrigeration system.

3. For the same refrigerating effect the size of the evaporator is smaller.

4. The required temperature of the evaporator can be achieved simply by adjusting the throttle valve of the same unit.

Demerits :

1. Initial cost is high.

2. The major disadvantages are inflammability, leakage of vapours and toxity. These have been overcome to a great extent by improvement in design.

Functions of Parts of a Simple Vapour Compression

System Here follows the brief description of various parts of a simple vapour compression system shown in Fig.

1. Compressor. The function of a compressor is to remove the vapour from the evaporator, and to raise its temperature and pressure to a point such that it (vapour) can be condensed with available condensing media.

2. Discharge line (or hot gas line). A hot gas or discharge line delivers the high-pressure, high-temperature vapour from the discharge of the compressor to the condenser.

3. Condenser. The function of a condenser is to provide a heat transfer surface through which heat passes from the hot refrigerant vapour to the condensing medium.

4. Receiver tank. A receiver tank is used to provide storage for a condensed liquid so that a constant supply of liquid is available to the evaporator as required.

5. Liquid line. A liquid line carries the liquid refrigerant from the receiver tank to the refrigerant flow control.

6. Expansion valve (refrigerant flow control). Its function is to meter the proper amount of refrigerant to the evaporator and to reduce the pressure of liquid entering the evaporator so that liquid will vapourize in the evaporator at the desired low temperature and take out sufficient amount of heat.

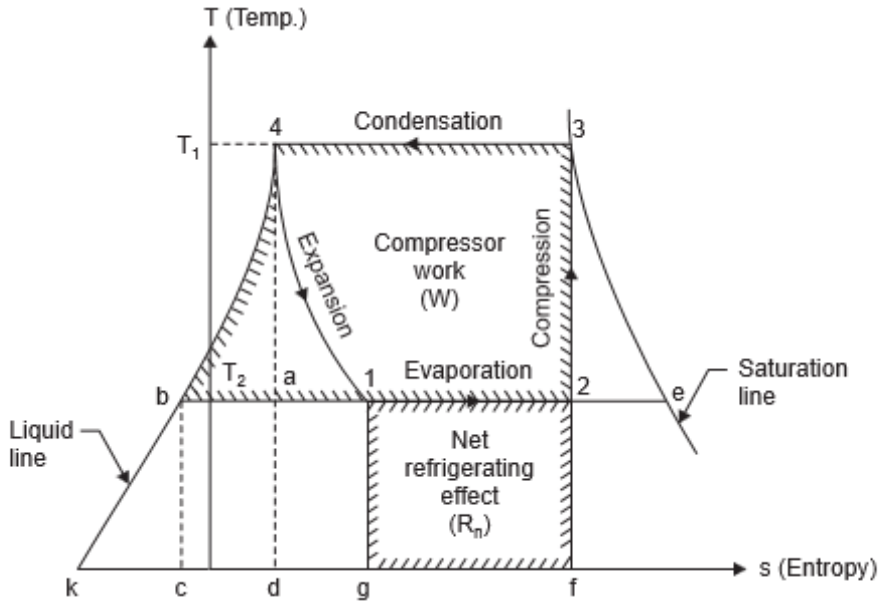
7. Evaporator. An evaporator provides a heat transfer surface through which heat can pass from the refrigerated space into the vapourizing refrigerant.

8. Suction line. The suction line conveys the low pressure vapour from the evaporator to the suction inlet of the compressor.

Vapour Compression Cycle on Temperature-Entropy (T-s) Diagram

We shall consider the following three cases :

1. **When the vapour is dry and saturated at the end of compression.** Fig.represents the vapour compression cycle, on T-s diagram the points 1, 2, 3 and 4 correspond to the state points 1, 2, 3 and 4 in Fig.



At point '2' the vapour which is at low temperature (T2) and low pressure enters the compressor's cylinder and is compressed adiabatically to '3' when its temperature increases to the temperature T1. It is then condensed in the condenser (line 3-4) where it gives up its latent heat to the condensing medium. It then undergoes throttling expansion while passing through the expansion valve and its again reduces to T2, it is represented by the line 4-1. From the T-s diagram it may be noted that due to this expansion the liquid partially evaporates, as its dryness fraction is represented by the ratio b_1/b_2. At '1' it enters the evaporator where it is further evaporated at constant pressure and constant temperature to the point '2' and the cycle is completed.

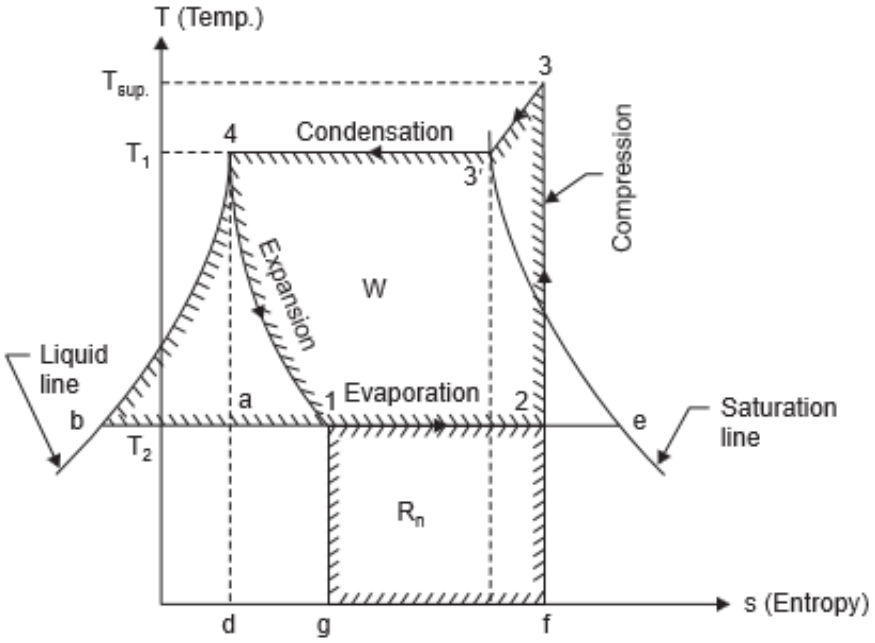
Work done by the compressor = W = Area '2-3-4-b-2'

Heat absorbed = Area '2-1-g-f-2'

$$\therefore C.O.P. = \frac{Heat extracted or refrigerating effect}{work done} = \frac{\text{Area '2 - 1 - g - f - 2'}}{\text{Area '2 - 3 - 4 - b - 2'}} \Rightarrow C.OP. = \frac{h_2 - h_1}{h_3 - h_2} = \frac{h_2 - h_4}{h_3 - h_2}$$

($h_1 = h_4$, since during the throttling expansion 4-1 the total heat content remains unchanged)

2. **When the vapour is superheated after compression.** If the compression of the vapour is continued after it has become dry, the vapour will be superheated, its effect on T-s diagram is shown in Fig. The vapour enters the compressor at condition '2' and is compressed to '3' where it is superheated to temperature Tsup. Then it enters the condenser. Here firstly superheated vapour cools to temperature T_1 (represented by line 3-3') and then it condenses at constant temperature along the line 3'-4 ; the remaining of the cycle ; however is the same as before.



Now, Work done = Area '2-3-3'-4-b-2'

and Heat extracted/absorbed = Area '2-1-g-f-2'

$$\therefore C.O.P. = \frac{Heatextracted}{workdone} = \frac{\text{Area '2-1-g-f-2'}}{\text{Area '2-3-3'-4-b-2'}} = \frac{h_2 - h_1}{h_3 - h_2}$$

In this case $h_3 = h_3' + cp\,(T_{sup.} - T_{sat.})$ and h_3' = total heat of dry and saturated vapour at the point '3''

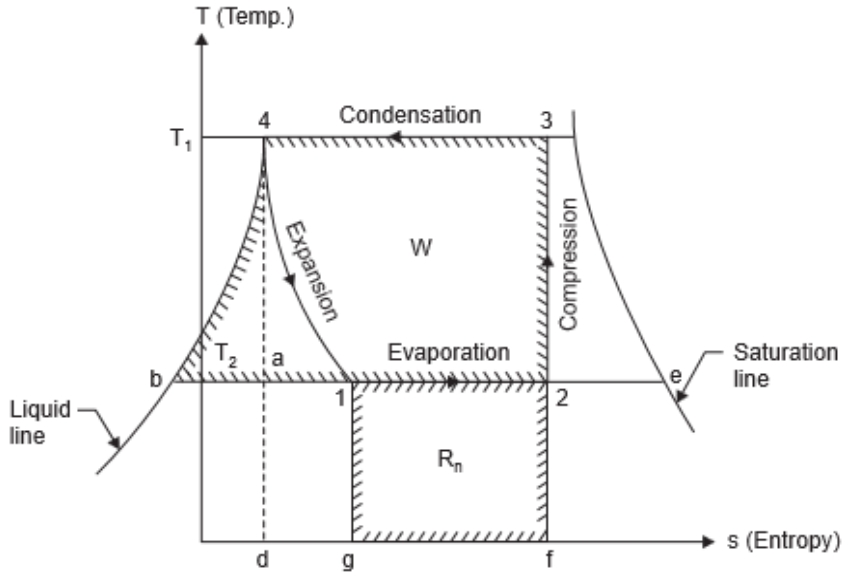
3. When the vapour is wet after compression.

. Work done by the compressor = Area '2-3-4-b-2'

Heat extracted = Area '2-1-g-f-2'

$$\therefore C.O.P. = \frac{Heatextracted}{workdone} = \frac{\text{Area '2-1-g-f-2'}}{\text{Area '2-3-4-b-2'}} = \frac{h_2 - h_1}{h_3 - h_2}$$

Note. If the vapour is not superheated after compression, the operation is called 'WET COMPRESSION' and if the vapour is superheated at the end of compression, it is known as 'DRY COMPRESSION'. Dry compression, in actual practice is always preferred as it gives higher volumetric efficiency and mechanical efficiency and there are less chances of compressor damage.



Factors Affecting the Performance of a Vapour Compression System

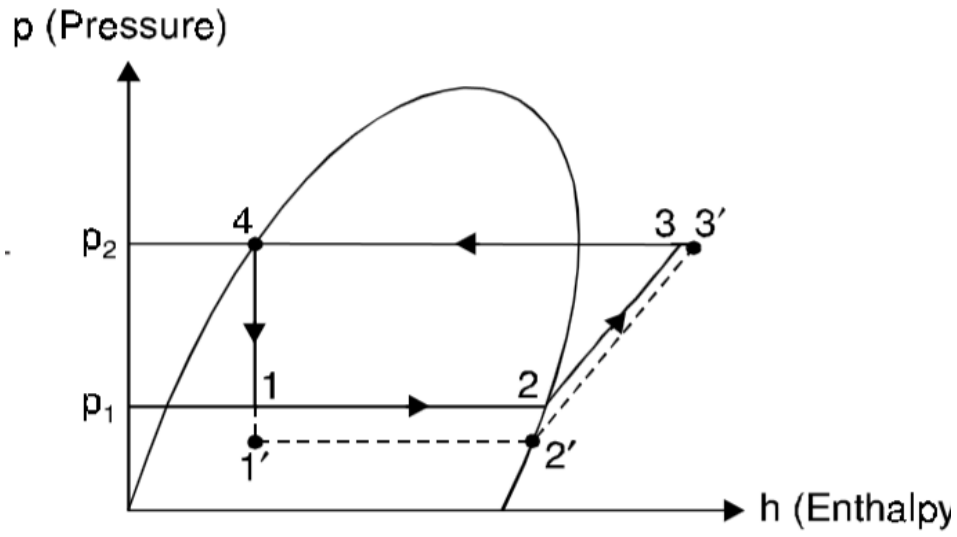
The factors which affect the performance of a vapour compression system are given below :

1. **Effect of suction pressure.** The effect of decrease in suction pressure is shown in Fig.

   The C.O.P. of the original cycle,

$$C.O.P. = \frac{h_2 - h_1}{h_3 - h_2}$$

   The C.O.P. of the cycle when suction pressure is decreased,

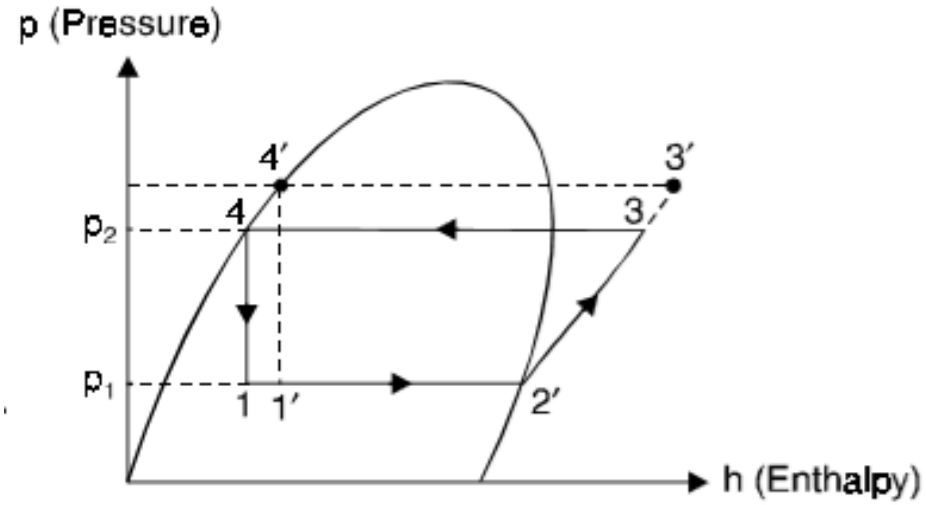


$$C.O.P. = \frac{h_2' - h_1'}{h_3' - h_2'} = \frac{(h_2 - h_1) - (h_2 - h_2')}{(h_3 - h_2) + (h_2 - h_2') + (h_3' - h_3)}$$

$$(\because h_1 = h_1')$$

This shows that the refrigerating effect is decreased and work required is increased. The net effect is to reduce the refrigerating capacity of the system (with the same amount of refrigerant flow) and the C.O.P.

2. **Effect of delivery pressure.** Fig. shows the effect of increase in delivery pressure.



C.O.P. of the original cycle,

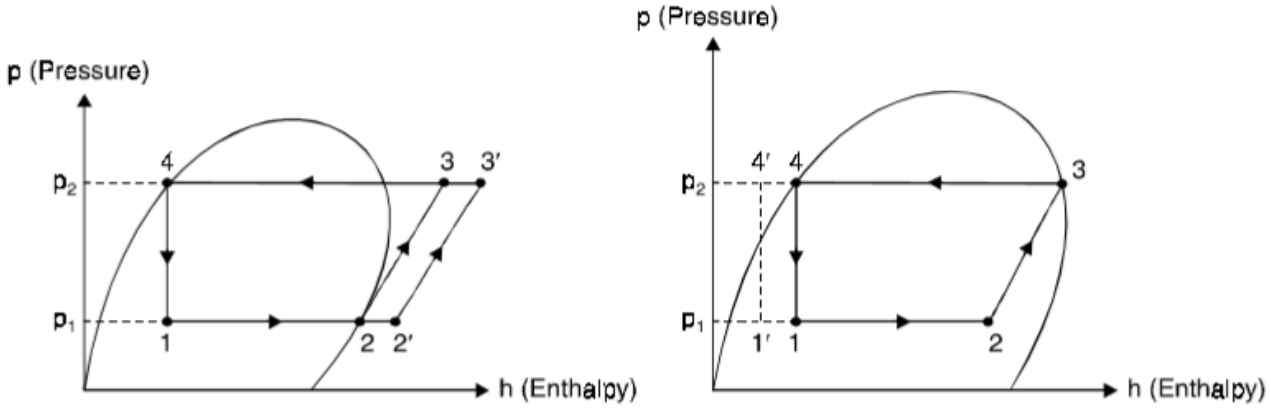
$$C.O.P. = \frac{h_2 - h_1}{h_3 - h_2}$$

C.O.P. of the cycle when delivery pressure is increased,

$$C.O.P. = \frac{h_2 - h_1'}{h_3' - h_2} = \frac{(h_2 - h_1) - (h_1' - h_1)}{(h_3 - h_2) + (h_3' - h_3)}$$

- The effect of increasing the delivery/discharge pressure is just similar to the effect of decreasing the suction pressure. The only difference is that the effect of decreasing the suction pressure is more predominant than the effect of increasing the discharge pressure.
- The following points may be noted : (i) As the discharge temperature required in the summer is more as compared with winter, the same machine will give less refrigerating effect (load capacity decreased) at a higher cost. (ii) The increase in discharge pressure is necessary for high condensing temperatures and decrease in suction pressure is necessary to maintain low temperature in the evaporator.

3.Effect of superheating. As may be seen from the Fig.the effect of superheating is to increase the refrigerating effect but this increase in refrigerating effect is at the cost of increase in amount of work spent to attain the upper pressure limit. Since the increase in work is more as compared to increase in refrigerating effect, therefore overall effect of superheating is to give a low value of C.O.P.



4. Effect of sub-cooling of liquid. 'Sub-cooling' is the process of cooling the liquid refrigerant below the condensing temperature for a given pressure. In Fig. the process of subcooling is shown by 4-4'. As is evident from the figure the effect of subcooling is to increase the refrigerating effect. Thus sub-cooling results in increase of C.O.P. provided that no further energy has to be spent to obtain the extra cold coolant required.

The sub-cooling or undercooling may be done by any of the following methods :

(i) Inserting a special coil between the condenser and the expansion valve.

(ii) Circulating greater quantity of cooling water through the condenser.

(iii) Using water cooler than main circulating water.

5.Effect of suction temperature and condenser temperature. The performance of the vapour compression refrigerating cycle varies considerably with both vapourising and condensing temperatures. Of the two, the vapourising temperature has far the greater effect. It is seen that the capacity and performance of the refrigerating system improve as the vapourising temperature increases and the condensing temperature decreases. Thus refrigerating system should always be designed to operate at the highest possible vapourising temperature and lowest possible condensing temperature, of course, keeping in view the requirements of the application.

Actual Vapour Compression Cycle

The actual vapour compression cycle differs from the theoretical cycle in several ways because of the following reasons :

(i) Frequently the liquid refrigerant is sub-cooled before it is allowed to enter the expansion valve, and usually the gas leaving the evaporator is superheated a few degrees before it enters the compressor. This superheating may occur as a result of the type of expansion control used or through a pick up of heat in the suction line between the evaporator and compressor.

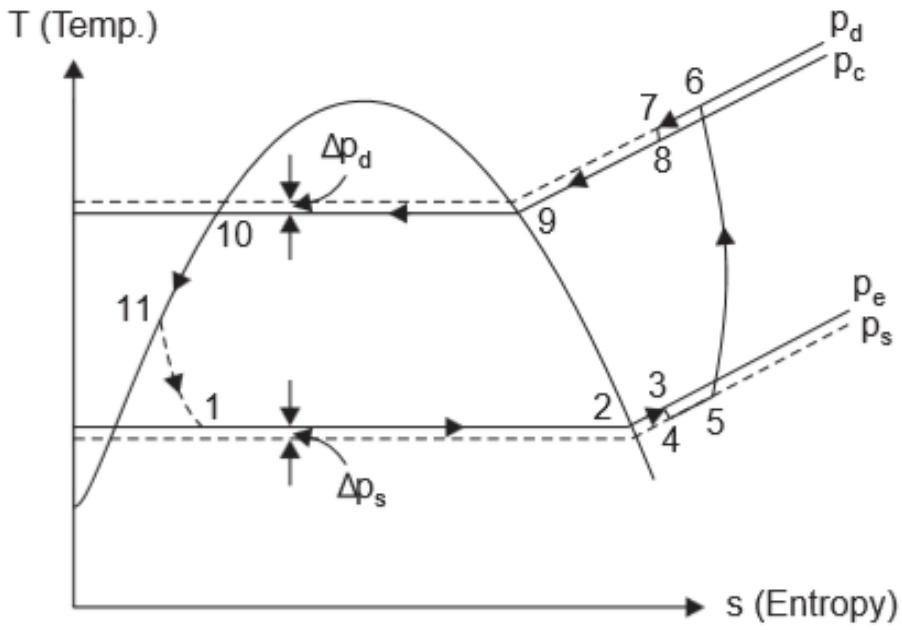
(ii) Compression, although usually assumed to be isentropic, may actually prove to be neither isentropic nor polytropic.

(iii) Both the compressor suction and discharge valves are actuated by pressure difference and this process requires the actual suction pressure inside the compressor to be slightly below that of the evaporator and the discharge pressure to be above that of condenser.

(iv) Although isentropic compression assumes no transfer of heat between the refrigerant and the cylinder walls, actually the cylinder walls are hotter than the incoming gases from the evaporator and colder than the compressed gases discharged to the condenser.

(v) Pressure drop in long suction and liquid line piping and any vertical differences in head created by locating the evaporator and condenser at different elevations.

Fig. shows the actual vapour compression cycle on T-s diagram. The various processes are discussed as follows :



Process 1-2-3. This process represents passage of refrigerant through the evaporator, with 1-2 indicating gain of latent heat of vapourisation, and 2-3, the gain of superheat before entrance to compressor. Both of these processes approach very closely to the constant pressure conditions (assumed in theory).

Process 3-4-5-6-7-8. This path/process represents the passage of the vapour refrigerant from entrance to the discharge of the compressor. Path 3-4 represents the throttling action that occurs during passage through the suction valves, and path 7-8 represents the throttling during passage through exhaust valves. Both of these actions are accompanied by an entropy increase and a slight drop in temperature. Compression of the refrigerant occurs along path 5-6, which is actually neither isentropic nor polytropic. The heat transfers indicated by path 4-5 and 6-7 occur essentially at constant pressure.

Process 8-9-10-11. This process represents the passage of refrigerant through the condenser with 8-9 indicating removal of superheat, 9-10 the removal of latent heat, and 10-11 removal of heat of liquid or sub-cooling.

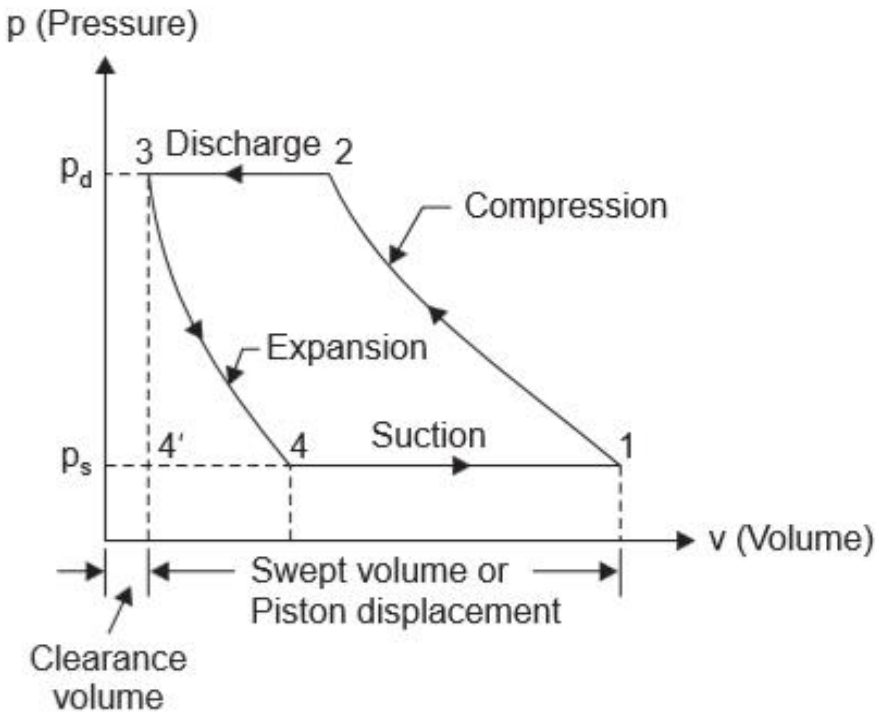
Process 11-1. This process represents passage of the refrigerant through the expansion valve, both theoretically and practically an irreversible adiabatic path.

Volumetric Efficiency

A compressor which is theoretically perfect would have neither clearance nor losses of any type and would pump on each stroke a quantity of refrigerant equal to piston displacement. No actual compressor is able to do this, since it is impossible to construct a compressor without clearance or one that will have no wire drawing through the suction and discharge valves, no superheating of the suction gases upon contact with the cylinder walls, or no leakage of gas past the piston or the valves. All these factors effect the volume of gas pumped or the capacity of the compressor, some of them affect the H.P. requirements per tonne of refrigeration developed.

'Volumetric efficiency' is defined as the ratio of actual volume of gas drawn into the compressor (at evaporator temperature and pressure) on each stroke to the piston displacement. If the effect of clearance alone is considered, the resulting expression may be termed clearance volumetric efficiency. The expression used for grouping into one constant all the factors affecting efficiency may be termed total volumetric efficiency.

Clearance volumetric efficiency. 'Clearance volume' is the volume of space between the end of the cylinder and the piston when the latter is in dead centre position. The clearance volume is expressed as a percentage of piston displacement. In Fig. the piston displacement is shown as 4'-1.



During the suction stroke 4'-1, the vapour filled in clearance space at a discharge pressure pd expands along 3-4 and the suction valve opens only when pressure has dropped to suction pressure ps, therefore actual volume sucked will be (v1 – v4) while the swept volume is (v1 – v4'). The ratio of actual volume of vapour sucked to the piston displacement is defined as clearance volumetric efficiency.

Thus,

Clearance volumetric efficiency, $\eta_{cv} = \frac{v_1 - v_4}{v_1 - v_4'} = \frac{v_1 - v_4}{v_1 - v_3} (\because v_4' = v_3)$

Considering polytropic expansion process 3-4, we have

$$p_s v_4^n = p_d v_3^n \Rightarrow \frac{p_d}{p_s} = \left(\frac{v_4}{v_3}\right)^n \Rightarrow v_4 = v_3 \left(\frac{p_d}{p_s}\right)^{\frac{1}{n}}$$

If the clearance ratio,

$$C = \frac{v_3}{v_1 - v_3} = \frac{clearancevolume}{sweptvolume} \Rightarrow \eta_{CV} = \frac{v_1 - v_4}{v_1 - v_3} = \frac{(v_1 - v_4') - (v_4 - v_4')}{(v_1 - v_3)} = \frac{(v_1 - v_3) - (v_4 - v_3)}{(v_1 - v_3)} (\because v_4' = v_3)$$

$$= 1 - \frac{v_4 - v_3}{v_1 - v_3} = 1 - \frac{v_3 \left(\frac{p_d}{p_s}\right)^{\frac{1}{n}} - v_3}{v_1 - v_3} = 1 + \frac{v_3}{v_1 - v_3} \left[1 - \left(\frac{p_d}{p_s}\right)^{\frac{1}{n}}\right] = 1 + C - C\left(\frac{p_d}{p_s}\right)^{\frac{1}{n}}$$

Hence clearance volumetric efficiency,

$$\eta_{CV} = 1 + C - C\left(\frac{p_d}{p_s}\right)^{\frac{1}{n}}$$

Total volumetric efficiency. The total volumetric efficiency (ηtv) of a compressor is best obtained by actual laboratory measurements of the amount of refrigerant compressed and delivered to the condenser. It is very difficult to predict the effects of wire-drawing, cylinder wall heating, and piston leakage to allow any degree of accuracy in most cases. The total volumetric efficiency can be approximately calculated if the pressure drop through the suction valves and the temperature of the gases at the end of the suction stroke are known and if it is assumed that there is no leakage past the piston during compression, it can be calculated (by modifying the eqn. above) by using the following equation

$$\eta_{CV} = 1 + C - C\left(\frac{p_d}{p_s}\right)^{\frac{1}{n}} x \frac{p_c}{p_s} x \frac{T_s}{T_c}$$

where the subscript 'c' refers to compressor cylinder and 's' refers to the evaporator or the suction line just adjacent to the compressor.

Mathematical Analysis of Vapour Compression Refrigeration

(i) **Refrigerating effect.** Refrigerating effect is the amount of heat absorbed by the refrigerant in its travel through the evaporator. In Fig. this effect is represented by the expression.

$Q_{evap.} = (h_2 - h_1)$ kJ/kg

In addition to the latent heat of vaporization it may include any heat of superheat absorbed in the evaporator.

(ii) **Mass of refrigerant.** Mass of refrigerant circulated (per second per tonne of refrigeration) may be calculated by dividing the amount of heat by the refrigerating effect.

∴ Mass of refrigerant circulated,

$$m = \frac{14000}{3600(h_2 - h_1)} kg/s - tonne$$

because one tonne of refrigeration means cooling effect of 14000 kJ/h.

(iii) **Theoretical piston displacement.** Theoretical piston displacement (per tonne of refrigeration per minute) may be found by multiplying the mass of refrigerant to be circulated (per tonne of refrigeration per sec.) by the specific volume of the refrigerant gas, (vg)2, at its entrance of compressor. Thus,

$$piston displacement_{theoretical} = \frac{14000}{3600(h_2 - h_1)}(v_g)_2 m^3/s - tonne$$

(iv) **Power (Theoretical) required.** Theoretical power per tonne of refrigeration is the power, theoretically required to compress the refrigerant. Here volumetric and mechanical efficiencies are not taken into consideration. Power required may be calculated as follows :

(a) When compression is isentropic :

Work of compression = $h_3 - h_2$

Power required = $m(h_3 - h_2)$ kW

where, m = Mass of refrigerant circulated in kg/s.

(b) When compression follows the general law pVn = constant :

$$Work of compression = \frac{n}{n-1}(p_3v_3 - p_2v_2) Nm/kg$$

$$power required = mx\frac{n}{n-1}(p_3v_3 - p_2v_2)x\frac{1}{10^3}kW(pi\sin N/m^2)$$

(v) **Heat rejected to compressor cooling water.** If the compressor cylinders are jacketed, an appreciable amount of heat may be rejected to the cooling water during compression. If the suction and discharge compression conditions are known, this heat can be determined as follows :

Heat rejected to compressor cooling water

$$= \left[\frac{n}{(n-1)}\left(\frac{p_3v_3 - p_2v_2}{1000}\right) - (h_3 - h_2)\right] kJ/kg$$

(vi) **Heat removed through condenser.** Heat removed through condenser includes all heat removed through the condenser, either as latent heat, heat of superheat, or heat of liquid. This is equivalent to the heat absorbed in the evaporator plus the work of compression.

∴ Heat removed through condenser = $m(h_3 – h_4)$ kJ/s (m = mass of refrigerant circulated in kg/s)

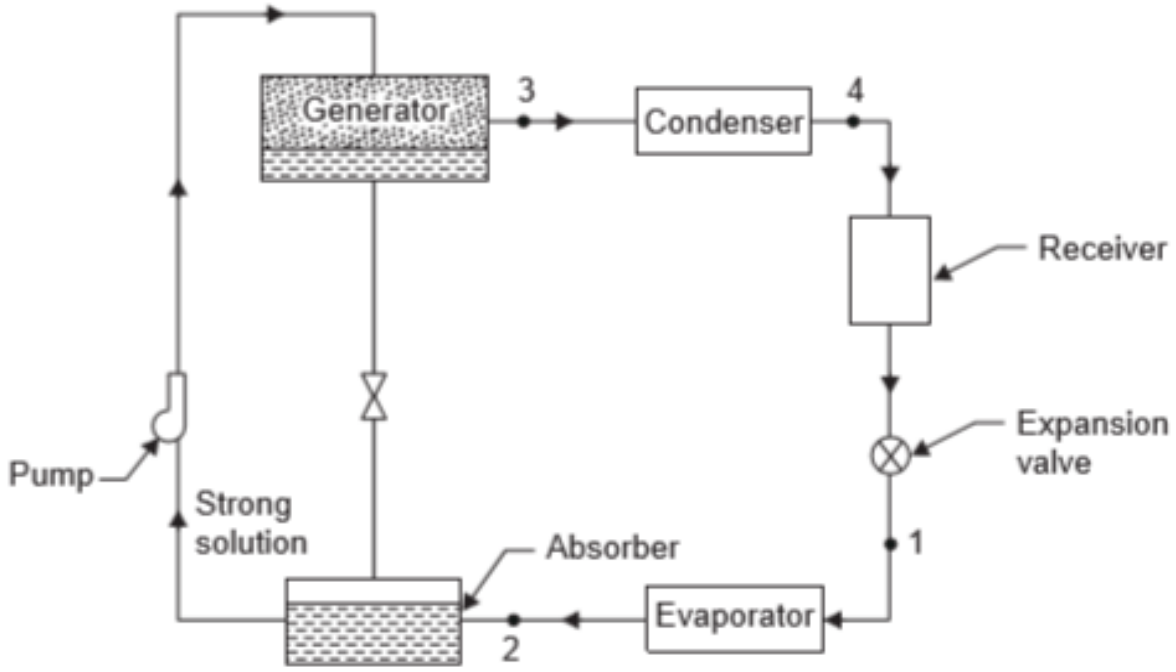
VAPOUR ABSORPTION SYSTEM

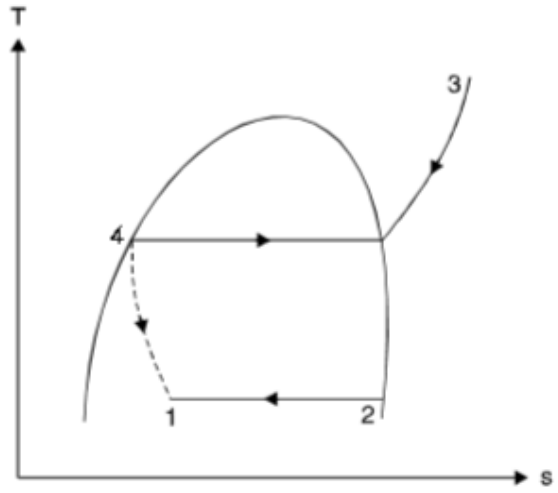
Introduction

In a vapour absorption system the refrigerant is absorbed on leaving the evaporator, the absorbing medium being a solid or liquid. In order that the sequence of events should be continuous it is necessary for the refrigerant to be separated from the absorbent and subsequently condensed before being returned to the evaporator. The separation is accomplished by the application of direct heat in a 'generator'. The solubility of the refrigerant and absorbent must be suitable and the plant which uses ammonia as the refrigerant and water as absorbent will be described.

Simple Vapour Absorption System

Refer Fig. below for a simple absorption system. The solubility of ammonia in water at low temperatures and pressures is higher than it is at higher temperatures and pressures. The ammonia vapour leaving the evaporator at point 2 is readily absorbed in the low temperature hot solution in the absorber. This process is accompanied by the rejection of heat. The ammonia in water solution is pumped to the higher pressure and is heated in the generator. Due to reduced solubility of ammonia in water at the higher pressure and temperature, the vapour is removed from the solution. The vapour then passes to the condenser and the weakened ammonia in water solution is returned to the absorber.

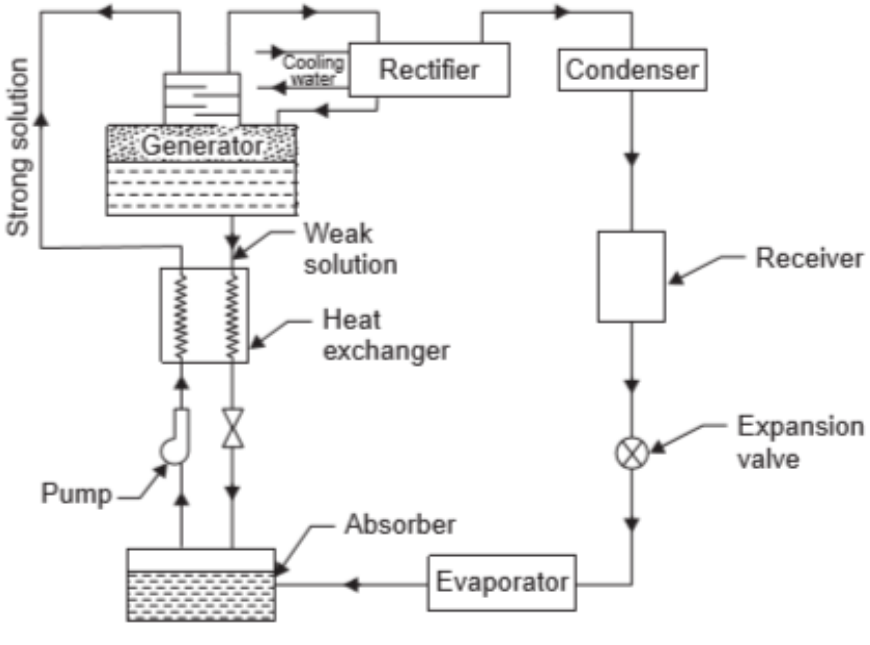




In this system the work done on compression is less than in vapour compression cycle (since pumping a liquid requires much less work than compressing a vapour between the same pressures) but a heat input to the generator is required. The heat may be supplied by any convenient form e.g. steam or gas heating.

Practical Vapour Absorption System

Refer Fig. below. Although a simple vapour absorption system can provide refrigeration yet its operating efficiency is low. The following accessories are fitted to make the system more practical and improve the performance and working of the plant.



1. Heat exchanger. 2. Analyser. 3. Rectifier.

1. Heat exchanger. A heat exchanger is located between the generator and the absorber. The strong solution which is pumped from the absorber to the generator must be heated ; and the weak solution from the generator to the absorber must be cooled. This is accomplished by a heat exchanger and consequently cost of heating the generator and cost of cooling the absorber are reduced.

2. Analyser. An analyser consists of a series of trays mounted above the generator. Its main function is to remove partly some of the unwanted water particles associated with ammonia vapour going to condenser. If these water vapours are permitted to enter condenser they may enter the expansion valve and freeze ; as a result the pipe line may get choked.

3. Rectifier. A rectifier is a water-cooled heat exchanger which condenses water vapour and some ammonia and sends back to the generator. Thus final reduction or elemination of the percentage of water vapour takes place in a rectifier.

The co-efficient of performance (C.O.P.) of this system is given by :

$$C.O.P. = \frac{\text{Heatextracted from the evaporator}}{\text{Heatsuppli ed in the generator Workdone by the liquid pump}}$$

Comparison between Vapour Compression and Vapour Absorption Systems

S. No	Particulars	Vapour compression system	Vapour absorption system
1	Type of energy supplied	Mechanical—a high grade	Mainly heat—a low grade energy
2	Energy supply	Low	High
3	Wear and tear	More	Less
4	Performance at part load	Poor	System not affected by variations of loads
5	Suitability	Used where high grade mechanical energy is available	Can also be used at remote places as it can work even with a simple kerosene lamp (of course in small capacities)
6	Charging of refrigerant	simple	Difficult
7	Leakage of refrigerant	More chances	No chance as there is no

			compressor or any reciprocating component to cause leakage.
8	**Damage**	**Liquid traces in suction line may damage the compressor**	**Liquid traces of refrigerant present in piping at the exit of evaporator constitute no danger**

Example. (i) What are the advantages of using an expansion valve instead of an expander in a vapour compression refrigeration cycle ?

(ii) Give a comparison between centrifugal and reciprocating compressors.

Solution.

(i) If an expansion cylinder is used in a vapour compression system, the work recovered would be extremely small, in fact not even sufficient to overcome the mechanical friction. It will not be possible to gain any work. Further, the expansion cylinder is bulky. On the other hand the expansion valve is a very simple and handy device, much cheaper than the expansion cylinder. It does not need installation, lubrication or maintenance.

The expansion valve also controls the refrigerant flow rate according to the requirement, in addition to serving the function of reducting the pressure of the refrigerant.

(ii) The comparison between centrifugal and reciprocating compressors :

The comparison between centrifugal and reciprocating compressors is given in the table below:

S. No	Particulars	Centrifugal compressor	Reciprocating compressor
1	Suitability	Suitable for handling large volumes of air at low pressures	Suitable for low discharges of air at high pressure.
2	Operational speeds	Usually high	Usually high
3	Air supply	Continuous	Pulsating

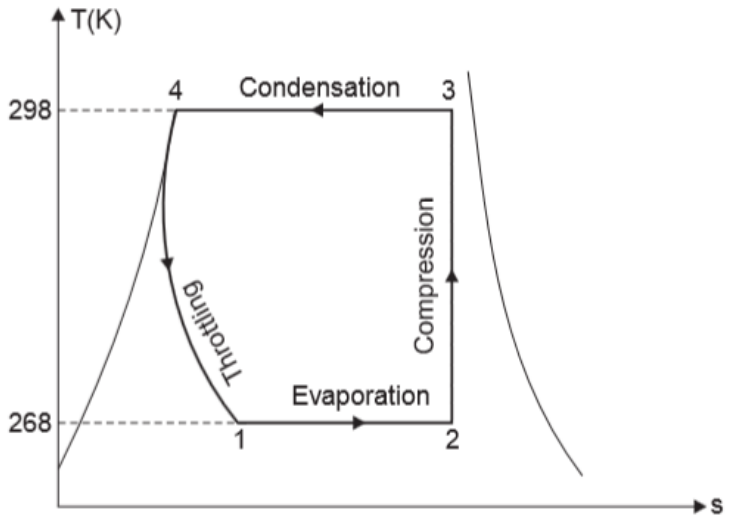
4	Balancing	Less vibrations	Cyclic vibrations occur
5	Lubrication system	Generally simple lubrication systems are required.	Generally complicated
6	Quality of air delivered	Air delivered is relatively more clean	Generally contaminated with oil.
7	Air compressor size	Small for given discharge	Large for same discharge
8	Free air handled	2000-3000 m^3/min	250-300 m^3/min
9	Delivery pressure	Normally below 10 bar	500 to 800 bar
10	Usual standard of compression	Isentropic compression	Isothermal compression
11	Action of compressor	Dynamic action	Positive displacement.

Example. A refrigeration machine is required to produce i.e., at 0°C from water at 20°C. The machine has a condenser temperature of 298 K while the evaporator temperature is 268 K. The relative efficiency of the machine is 50% and 6 kg of Freon-12 refrigerant is circulated through the system per minute. The refrigerant enters the compressor with a dryness fraction of 0.6. Specific heat of water is 4.187 kJ/kg K and the latent heat of ice is 335 kJ/kg. Calculate the amount of ice produced on 24 hours. The table of properties of Freon-12 is given below :

Temperature K	Liquid heat kJ/kg	Latent heat kJ/g	Entropy of liquid kJ/kg
298	59.7	138.0	0.2232
268	31.4	154.0	0.1251

Solution. Given : m = 6 kg/min. ; $\eta_{relative}$ = 50% ; x2 = 0.6 ; c_{pw} = 4.187 kJ/kg K ; Latent heat of ice = 335 kJ/kg.

h_{f2} = 31.4 kJ/kg ; h_{fg2} = 154.0 kJ/kg ; h_{f3} = 59.7 kJ/kg ; h_{fg3} = 138 kJ/kg ; h_{f4} = 59.7 kJ/kg ...From the table given above $h_2 = h_{f_2} + xh_{fg_2} = 31.4 + 0.6X154 = 123.8kJ/kg$



For isentropic compression 2-3, we have

$s_3 = s_2$

$$s_{f_3} + x_3 \frac{h_{fg_3}}{T_3} = s_{f_2} + x_2 \frac{h_{fg_2}}{T_2} \Rightarrow 0.2232 + x_3 X \frac{138}{298} = 0.1251 + 0.6X \frac{154}{268} = 0.4698$$

$$\therefore x_3 = (0.4698 - 0.2232) X \frac{298}{138} = 0.5325$$

$$Now, h_3 = h_{f_3} + x_3 h_{fg_3} = 59.7 + 0.5325X138 = 133.2kJ/kg$$

$$Also, h_1 = h_{f_4} = 59.7kJ/kg$$

$$TheoreticalC.O.P. = \frac{R_n}{W} = \frac{h_2 - h_1}{h_3 - h_2} = \frac{123.8 - 59.7}{133.2 - 123.8} = 6.82$$

Actual C.O.P. = $\eta_{relative} \times (C.O.P.)_{theoretical} = 0.5 \times 6.82 = 3.41$

Heat extracted from 1 kg of water at 20°C for the formation of 1 kg of ice at 0°C

= 1 × 4.187 × (20 – 0) + 335 = 418.74 kJ/kg

Let m_{ice} = Mass of ice formed in kg/min.

$$(C.O.P.)_{actual} = 3.41 = \frac{R_n(actual)}{W} = \frac{m_{ice} X\, 418.74}{m(h_3 - h_2)} \Rightarrow m_{ice} = \frac{6(133.2 - 123.8) X\, 3.41}{418.74} = 0.459 kg/\min$$

$$= \frac{0.459 X\, 60 X\, 24}{1000} tonnes(in 24 hours) = 0.661 tonnes$$